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Designing Doctor-Patient Shared Decision-Making Processes under Bounded Rationality

Abstract. Problem Definition: Shared decision-making (SDM) processes, in which doctors and patients work together to choose among treatment options, have gained increasing support from clinicians and governments. Despite the recognized benefits and limitations of these processes, clear guidelines on when and how to implement them are lacking.

Methodology/Results: We develop a stylized analytical model to derive a set of results characterizing when and how to personalize the treatment decision-making process between doctors and patients, taking into account the bounded rationality of doctors and patients. For example, we find that personalizing treatment to account for individual patient preferences is beneficial when treatments have a large expected medical difference across patient types relative to the degree of patient decision error. We also find that the benefits of personalizing treatment to account for individual medical prognoses depends on the degree of patient decision participation. With patient participation, prognosis personalization can even be detrimental when patients strongly prefer one treatment over the other.

Managerial implications: Challenging common medical belief that advocates for uniform increases in personalization, we show that limitations in medical prognoses accuracy and human cognition imply the existence of trade-offs between personalization and standardization on multiple dimensions within doctor-patient SDM processes. Our results prescribe whether and how to target personalization efforts based on environmental factors, doctor, and patient characteristics.

Key words: shared decision-making, treatment personalization, standardization, bounded rationality

1. Introduction

Healthcare systems are increasingly moving toward patient-centered care, which emphasizes tailoring treatment plans to individual patient needs and preferences through personalization (Breen et al. 2010). This approach involves personalizing treatments based on two key dimensions: medical prognoses and patient preferences. The former refers to informing patients about their personalized med-

ical prognosis regarding risks and outcome predictions, while the latter refers to integrating patients' lifestyle and risk preferences into treatment decisions by actively involving them in the medical decision-making process (Rogowski et al. 2015). This approach is often referred to as shared decision-making (SDM) in the healthcare literature. For example, Coulter and Collins (2011) define SDM as "a process in which clinicians and patients work together to select tests, treatments, management or support packages, based on clinical evidence and the patient's informed preferences". This collaborative model ensures that treatment decisions align with each patient's unique needs and preferences (Daack-Hirsch and Campbell 2014, Bagshaw et al. 2021).

In contrast, standardized medicine (SM) refers to the systematic application of evidence-based protocols and guidelines to ensure uniformity in healthcare delivery, adopting a one-size-fits-all approach. These protocols, typically developed through randomized clinical trials (RCTs), focus on the "average patient" rather than the individual patient (Romana 2006, de Leon 2012). Proponents of SDM criticize SM for neglecting the individual patient, calling it a "doctor-centered" approach (Sweeney et al. 1998) that overemphasizes the disease and neglects the subjective needs and desires of the patient (Haines et al. 2019, Spatz et al. 2017, JM 2018). Such criticisms have driven initiatives like the United Kingdom's National Health Service (NHS)'s motto, "no decision about me without me," and the acknowledgement that SDM is appropriate in most healthcare situations that involve preference-sensitive decisions (NHS England and NHS Improvement 2019). The U.S. Patient Protection and Affordable Care Act similarly promotes SDM (ACA 2010). In 2011, 58 experts from 18 countries published the Salzburg Statement on Shared Decision Making, calling for clinicians and patients to use SDM (Salzburg Global Seminar 2011).

Although many experts advocate SDM, doctors often cite challenges in its implementation and are sometimes reluctant to use it (Légaré and Witteman 2013). One major barrier is patients' lack of information literacy. For example, a medical oncologist observed, "Sometimes I feel like if we lay all the options out there sometimes it confuses them and they are not really making a good decision in the end" (Zeuner et al. 2015). Another noted, "the main thing that stands in my way [of using SDM] is the patient's inability to understand risk." (Schoenfeld et al. 2019). Additionally, doctors often report confusion or uncertainty about when SDM should be applied (Baghus et al. 2022, van der Horst et al. 2022, Barker et al. 2019). The use of SDM varies greatly among doctors, even within the same hospital network. While one doctor might routinely involve patients in every decision, stating, "Clinical practice is about patient education and shared decision making," another might base decisions solely on evidence, saying, "I have a manual that I rely on" (Alameddine et al. 2020). Moreover, factors such

as experience, confidence, and awareness of their limitations also influence how frequently SDM is implemented (Simmons et al. 2016, Schoenfeld et al. 2018, 2019, Waddell et al. 2021).

In this paper, we develop a stylized analytical model to derive guidelines for when and how systems should design doctor-patient shared decision making processes. We explicitly incorporate *both* the potential benefits claimed by its advocates *and* the real-world challenges that hinder its use. These challenges include the cognitive limitations and behavioral tendencies experienced by both doctors and patients. Specifically, we ask the following research questions: (1) How do patient cognitive limitations affect the optimal personalized medical prognoses communication? (2) Under what conditions should systems have doctors personalize medical diagnoses?, and (3) Under what conditions should systems personalize treatment decisions based on patient preferences?

To address these questions, we consider a doctor-patient decision process with two possible treatments. We develop a stylized model in which patients differ in both their medical prognoses under each treatment and their preferences for these medical prognoses. Doctors have private information about patients' medical prognoses, and patients have private information about their preferences about medical prognoses. Our model incorporates the concept of bounded rationality, acknowledging that doctors have noisy signals about patient prognoses and that patients may make errors when they try to apply their preferences to the prognoses that they are given.

	No patient participation	Patient participation
Standardized medical prognoses	Standardized Medicine (SM)	Patient Choice (PC)
Personalized medical prognoses	Personalized Medicine (PM)	Coproduction (CP)

Table 1 Prognosis-to-treatment decision-making processes.

We use our model to compare the performances of the four types of decision-making processes described in Table 1. These processes vary based on the degree of personalization across two key dimensions. In medical prognosis personalization, the doctor provides patients with tailored information about their specific risks and predicted outcomes. In patient participation, the doctor integrates patients' personal lifestyles, values, and risk preferences into medical decision-making by allowing them to actively participate in the decision-making process.

We now outline the structure of the paper and preview our main results. We review the related literature in §2 and introduce our model in §3. In §4, we establish how doctors *should* respond to cognitive limitations under PM and CP. When providing personalized medical prognosis information, doctors combine prior knowledge with private, yet potentially noisy, signals of patient prognoses. A key performance issue here is whether doctors have the metacognition to make adjustments to account

for their noisy signals (i.e., by using Bayesian weights). Specifically, under PM, we find that doctors should place Bayesian weights on the priors to account for their random errors accurately. However, with patient errors introduced under CP, Bayesian weights are no longer optimal. In these cases, depending on the strength of the patient's treatment preferences, doctors should adjust the weight on priors—either increasing or decreasing it relative to the Bayesian weight.

In §5, we identify when clinical guidelines for SDM should advise doctors to personalize medical prognoses. Without patient participation, the decision to switch from SM to PM depends heavily on how well calibrated doctors in accounting for their noisy prognoses signals (i.e., how Bayesian they are). Specifically, we find that switching from SM to PM is always beneficial for Bayesian doctors or those who rely less (relative to Bayesian doctors) on noisy private information. However, it could be detrimental for doctors who weigh this private information more heavily than their Bayesian counterparts. On the other hand, with patient participation, the decision to switch from PC to CP depends on the strength of patient preferences, rather than whether the doctor is Bayesian. More specifically, when patients have weak preferences between treatments, we show that it is always beneficial to switch from PC to CP. However, when patients have strong preferences for a particular treatment, switching from PC to CP could be detrimental. Interestingly, in such cases, prognosis personalization may lead to a utility loss even for Bayesian doctors. This is because when boundedly rational patients with strong preferences are involved in decision-making, Bayesian weights are no longer optimal. In particular, Bayesian doctors "overweight" the signal relative to the optimal weight, which worsens the performance of prognosis personalization.

In §6, we establish when clinical guidelines for SDM should advise doctors to allow patient participation. We show that it is beneficial to allow patient participation (i.e., switch from SM to PC, or from PM to CP) when the mean difference in treatment effects between heterogeneous patient types is sufficiently large relative to patient error.

In §7, we synthesize the insights from the previous sections to provide guidance on the optimal design of prognosis-to-treatment decision-making processes, identifying when SM, PM, PC, or CP is superior. The decision tree in Figure 2 in §7 summarizes these findings. We conclude in §8 with a discussion of managerial insights, limitations, and directions for future research.

2. Related Literature

Patient Preference and Prognosis Personalization in the Medical Literature: The medical literature largely supports personalizing treatment based on both medical prognoses and patient preferences (Braddock III et al. 1999, Oshima Lee and Emanuel 2013, Veroff et al. 2013, Daack-Hirsch and

Campbell 2014, Shay and Lafata 2015). However, some studies cite patients' lack of health literacy as a significant barrier to effective SDM and advise caution when involving patients with low health literacy in SDM (McCaffery et al. 2010, Shippee et al. 2015, Palumbo and Manna 2018). Unfortunately, there are no clear guidelines on how to operationalize SDM—specifically, on when to personalize treatments to account for medical prognoses and/or patient preferences (van Veenendaal et al. 2018, Barker et al. 2019, Baghus et al. 2022, van der Horst et al. 2022), leaving such decisions highly dependent on clinician judgment.

While the existing literature primarily focuses on a dichotomy between standardized medicine (no personalization) and coproduction (full personalization), this paper offers a more nuanced perspective. In particular, in addition to standardized medicine and coproduction, we explore intermediate processes—personalized medicine and patient choice—that allow for personalizing treatment based on either medical prognoses or patient preferences. In doing so, we contribute to this stream of literature by providing guidelines for when full personalization (coproduction), no personalization (standardized medicine), or partial personalization (personalized medicine or patient choice) is most appropriate.

Finally, while the medical literature frequently cites patients' lack of health literacy as a barrier to SDM, it often overlooks doctors' limitations (McCaffery et al. 2010, Shippee et al. 2015, Palumbo and Manna 2018). A key contribution of this paper is to consider not only patient errors but also doctor errors when developing guidelines for personalizing treatment to account for individual medical prognoses and patient preferences.

Patient Preference and Prognosis Personalization in the Healthcare Operations Management Literature: Patient participation in care decisions and the incorporation of patient preferences have been modeled in the healthcare operations management literature. For example, Ahn and Hornberger (1996) consider incorporating patient preferences for health states into the allocation process for cadaveric kidney transplants, while Batun et al. (2018) consider incorporating patient preferences for risk into liver acceptance decisions for patients with end-stage liver disease. Ayvaci et al. (2018) develop a modeling framework that incorporates patient risk preferences into diagnostic decisions following mammography screening. In optimizing decisions about whether and when to perform biopsies for patients on active surveillance for prostate cancer, Li et al. (2023) allow the weighting of reward criteria to vary according to patient preferences.

In addition to treatment personalization based on patient preferences, a growing body of research focuses on tailoring treatment plans to individual patient prognoses and risk profiles. For example, Ibrahim et al. (2016) design a partially observable Markov decision process (POMDP) to customize

anticoagulation therapy based on the patient's individual response and sensitivity to treatment, with the goal of minimizing stroke risk. Zargoush et al. (2018) determine the optimal sequence of antihypertensive treatments by considering individual risk factors such as age, gender, and smoking habits. Chen et al. (2021) introduce a decision support system to customize radiation treatment plans based on predicted individual outcomes. Hajjar and Alagoz (2023) develop a framework for personalized screening decisions that take into account patients' comorbidities such as diabetes and hypertension.

All of these papers assume that decision-makers are perfectly rational, whereas we allow for doctors and patients to be boundedly rational.

Co-production in the Service Operations Management Literature: Co-production in service systems refers to customers playing an active role in the creation of the final output. This concept has received considerable attention in the service operations management literature (Fuchs et al. 1968, Sampson and Froehle 2006), with many studies analyzing its implications using analytical models. For example, Xue and Field (2008) consider a co-production process with information stickiness in consulting services, focusing on work allocation between consultant and client and pricing decisions. Roels (2014) identifies the optimal design of a co-production process between a customer and a service provider by investigating how much interaction is needed. They find that as a task becomes less standardized, it is optimal to increase the interaction between the customer and the service provider. Daw et al. (2020) develop new stochastic models for service co-production in contact centers by incorporating dynamic factors that depend on the mechanics of the interaction, such as the number of words written by each party. Some studies have examined the optimal contract design for service co-production. For example, Rahmani et al. (2017) consider a knowledge-intensive project that requires the involvement of both the client and the vendor. They provide several insights into the optimal contract design when the client cannot monitor and verify the vendor's efforts.

Bounded Rationality in Behavioral Operations Management and Judgment and Decision Making: Several papers in behavioral operations management consider the role of bounded rationality in the design of operational systems (Simon 1957). For example, researchers have examined how bounded rationality impacts the optimal design of supply chain contracts (Ho and Zhang 2008, Kalkanci et al. 2011, Su 2008), queues (Huang et al. 2013), auctions (Davis et al. 2014) and forecasting processes (Kremer et al. 2016, Tong and Feiler 2017, Ibrahim et al. 2021) in the presence of human random error. Similarly, this paper examines the role of random errors of doctors and patients on the design of SDM.

3. Model Setting

In this section, we introduce our modeling framework. In §3.1, we describe the utility functions of the patients. In §3.2, we present the four different prognosis-to-treatment decision-making processes we consider in the paper. In §3.3, we model the cognitive limitations of both doctors and patients.

3.1. Patients' Utility Functions

A doctor and a patient need to decide between two treatments: A and B. These treatments differ in their risks and benefits along two key dimensions, x and y. We consider two types of patients, called type-1 and type-2, who differ in the weight they place on each dimension. The utility of treatment t for a type-i patient is given by:

$$U_{it} = w_i X_t + (1 - w_i) Y_t. \tag{1}$$

Here, X_t and Y_t represent the medical responses of patients to treatment t, indicating the true medical *prognosis* along the x and y dimensions, respectively. The parameter $w_i \in [0, 1]$ captures the importance of the x dimension for a type-i patient.

We make the following distributional assumptions for analytical purposes: X_t and Y_t are normally distributed with means μ_{Xt} and μ_{Yt}, and variances σ^2_{Xt} and σ^2_{Yt}, respectively.[1] For algebraic convenience, we assume that $\sigma^2_{XA} = \sigma^2_{XB} = \sigma^2_X/2$ and $\sigma^2_{YA} = \sigma^2_{YB} = \sigma^2_Y/2$, though our results hold without this condition. An arriving patient is a type-1 patient with probability p and a type-2 patient with probability $1 - p$. Hence, the weight on the x dimension for the average patient is

$$\bar{w} = pw_1 + (1 - p)w_2. \tag{2}$$

The utility difference between treatments A and B for the average type-1 and type-2 patient are:

$$\Delta\mu_1 = w_1(\mu_{XA} - \mu_{XB}) + (1 - w_1)(\mu_{YA} - \mu_{YB}), \tag{3}$$

$$\Delta\mu_2 = w_2(\mu_{XA} - \mu_{XB}) + (1 - w_2)(\mu_{YA} - \mu_{YB}). \tag{4}$$

Let $|\Delta\mu_1|$ and $|\Delta\mu_2|$ measure the absolute difference in utility between treatment A and treatment B for the average type-1 and type-2 patient, respectively. As these values increase, the average patient's utility for a given treatment becomes more distinct. As they approach zero, patients become more neutral toward the two treatments.

Without loss of generality, we make the following assumptions:

[1] We will discuss the necessity of assuming a normal distribution for X_t and Y_t in Section 3.3.

- Type-1 patients place a higher weight on the x dimension than type-2 patients, i.e., $w_1 > w_2$.
- For the average patient, treatment A is "better" on the x dimension, while treatment B is "better" on the y dimension, i.e., $\mu_{XA} > \mu_{XB}$ and $\mu_{YA} < \mu_{YB}$.
- Treatment A is better for the average type-1 patient, while treatment B is better for the average type-2 patient, i.e., $\Delta\mu_1 > 0$ and $\Delta\mu_2 < 0$.
- Treatment B is better for the overall average patient, i.e., $\bar{w}(\mu_{XA}-\mu_{XB})+(1-\bar{w})(\mu_{YA}-\mu_{YB}) < 0$.

In general, doctors are considered experts in making medical prognoses, while patients are experts in their preferences (Ng and Lee 2021, p.4). Therefore, we assume that patients know their type (type-1 or type-2), but doctors do not. Conversely, we assume that doctors observe the realizations of X_t and Y_t for a given patient, but patients do not. The following two examples help conceptualize this utility model.

Example 1 *Consider two treatment options for lung cancer: radiation (treatment A) and surgical removal (treatment B). Radiation may offer a lower survival time but is usually easier and less painful. Let X_A and X_B denote the ease of treatments A and B, respectively, with $\mu_{XA} - \mu_{XB} > 0$ since radiation is generally easier. Similarly, let Y_A and Y_B represents the natural logarithm of survival time under treatments A and B, respectively, with $\mu_{YA} - \mu_{YB} < 0$ since surgical removal is generally associated with longer survival. Lastly, w_1 and w_2 capture the importance placed on ease of treatment (relative to survival time) by type-1 and type-2 patients, respectively.*

Example 2 *Consider two drug options for depression: Venlafaxine (Treatment A) and Mirtazapine (Treatment B).*[2] *Venlafaxine generally leads to less weight gain than Mirtazapine but is harder to discontinue. Here, X_A and X_B denote the ease of keeping the weight constant, with $\mu_{XA} - \mu_{XB} > 0$. Y_A and Y_B denote the ease of discontinuing the drugs, with $\mu_{YA} - \mu_{YB} < 0$. Finally, w_1 and w_2 represent the importance placed on the ease of keeping the weight constant versus ease of discontinuation by type-1 and type-2 patients, respectively.*

3.2. Prognosis-to-Treatment Decision-Making Processes

Under a prognosis-to-treatment decision-making process, given the medical prognoses $\hat{X}_t = \hat{x}_t$ and $\hat{Y}_t = \hat{y}_t$ developed by the doctor, and the patient's perceived preferences ($\hat{w}_i$), the decision maker—who may be either the doctor or the patient, depending on the process—solves the following optimization problem to select a treatment for a patient of type-i:

$$\max_{t\in\{A,B\}} \hat{U}_{it}(\hat{w}_i, \hat{x}_t, \hat{y}_t). \tag{5}$$

[2] Mayo Clinic's decision aid for this choice is available at https://depressiondecisionaid.mayoclinic.org/app/depression.

Here, $\hat{U}_{it}(\hat{w}_i, \hat{x}_t, \hat{y}_t)$ represents the utility of a type-i patient under treatment t, as perceived by the decision maker. The exact expression of this utility function will be provided below as we introduce our four prognosis-to-treatment decision-making processes.

- **Standardized Medicine (SM):** The doctor selects a treatment based on average patient prognoses and weights without involving patients in the decision-making. Hence, letting $\hat{w}_i = \bar{w}$, $\hat{x}_t = \mu_{Xt}$ and $\hat{y}_t = \mu_{Yt}$ in (5), the utility of treatment t perceived by the doctor under SM is equal to:

$$\hat{U}_{it}(\bar{w}, \mu_{Xt}, \mu_{Yt}) = \bar{w}\mu_{Xt} + (1-\bar{w})\mu_{Yt}. \tag{6}$$

  Since we assume $\bar{w}(\mu_{XA} - \mu_{XB}) + (1-\bar{w})(\mu_{YA} - \mu_{YB}) < 0$, SM is equivalent to always choosing treatment B in our model.

- **Personalized Medicine (PM):** The doctor makes personalized predictions about the prognoses for each treatment, $\hat{X}_t = \hat{x}_t$ and $\hat{Y}_t = \hat{y}_t$, and decides based on the average patient weights.[3] Letting $\hat{w}_i = \bar{w}$ in (5), the utility of treatment t perceived by the doctor under PM is equal to:

$$\hat{U}_{it}(\bar{w}, \hat{x}_t, \hat{y}_t) = \bar{w}\hat{x}_t + (1-\bar{w})\hat{y}_t. \tag{7}$$

- **Patient Choice (PC):** The doctor provides average prognosis values (μ_{Xt} and μ_{Yt}) to the patient; who then makes a decision based on their preferences (w_i). However, while merging their preferences with the medical prognoses, the patient adds a random error γ_{it} due to their misinterpretation of the information shared by the doctor.[4] Letting $\hat{w}_i = w_i$, $\hat{x}_t = \mu_{Xt}$ and $\hat{y}_t = \mu_{Yt}$ in (5), a type-i patient perceives the utility of treatment t under PC to be

$$\hat{U}_{it}(w_i, \mu_{Xt}, \mu_{Yt}) = w_i\mu_{Xt} + (1-w_i)\mu_{Yt} + \gamma_{it}. \tag{8}$$

- **Coproduction (CP):** Both the doctor and the patient participate in decision-making. The doctor provides personalized medical prognosis predictions, $\hat{X}_t = \hat{x}_t$ and $\hat{Y}_t = \hat{y}_t$, and the patient decides based on their preferences (w_i).[3] Similar to PC, the patient under CP adds a random error γ_{it} while deciding between the two treatments. Letting $\hat{w}_i = w_i$ in (5), a type-i patient perceives the utility of treatment t under CP to be:

$$\hat{U}_{it}(w_i, \hat{x}_t, \hat{y}_t) = w_i\hat{x}_t + (1-w_i)\hat{y}_t + \gamma_{it}. \tag{9}$$

[3] The personalized medical prognosis predictions, $\hat{X}_t$ and $\hat{Y}_t$, will be introduced precisely in (11) in Section 3.3.

[4] The details about patient random error term γ_{it} will be introduced precisely in Section 3.3.

The optimization problem in (5) specifies which treatment is selected for a given patient under a prognosis-to-treatment process. To compare the performances of SM, PM, PC, and CP, we calculate the expected utilities of these four processes by averaging the actual utilities of the selected treatments across the entire patient population. Throughout the paper, we use EU^{SM}, EU^{PM}, EU^{PC}, and EU^{CP} to represent the expected utilities of SM, PM, PC, and CP, respectively. See Appendix A for the exact expressions of EU^{SM}, EU^{PM}, EU^{PC}, and EU^{CP}.

3.3. Doctor and Patient Informational and Cognitive Limitations

In practice, neither doctors nor patients are perfectly rational decision makers. Doctors face imperfect information and their judgments about a patient's prognosis may contain random errors (Gigerenzer and Muir Gray 2011, Kahneman et al. 2016). Patients, too, may struggle to interpret medical information and combine it with their personal preferences, often due to limitations like illiteracy and innumeracy (Williams et al. 2002).

We begin by describing how doctors develop personalized prognosis predictions under PM and CP. Doctors observe noisy signals S_t^X and S_t^Y about the prognosis on the x and y dimensions, given by:

$$S_t^X := X_t + \mathcal{E}_t^X, \qquad S_t^Y := Y_t + \mathcal{E}_t^Y, \tag{10}$$

where $\mathcal{E}_t^X$'s and $\mathcal{E}_t^Y$'s are identically normally distributed random variables with mean zero and variances $\sigma_{d,X}^2/2$ and $\sigma_{d,Y}^2/2$, respectively. Here, $\sigma_{d,X}$ and $\sigma_{d,Y}$ capture sources of doctor prognoses errors including imperfect information as well as inconsistencies in doctor's judgments. The normal distribution assumption for the errors as well as the prior beliefs (i.e., true medical prognoses X_t and Y_t in this paper) is common in behavioral operations management and information-updating literature (Grossman and Stiglitz 1976, 1980, Morris and Shin 2002, Allen et al. 2006, Feiler and Tong 2022). We assume that $\mathcal{E}_t^X$'s and $\mathcal{E}_t^Y$'s are independent across treatments, dimensions, and patient, and also independent of X_t's and Y_t's.

The doctor's predictions for each prognosis dimension, $\hat{X}_t$ and $\hat{Y}_t$, are then weighted combinations:

$$\hat{X}_t = \lambda_X S_t^X + (1-\lambda_X)\mu_{Xt}, \quad \hat{Y}_t = \lambda_Y S_t^Y + (1-\lambda_Y)\mu_{Yt}, \tag{11}$$

where $\lambda_X, \lambda_Y \in [0,1]$ indicate the doctor's weight on the signal. Here, the assumption of a normal distribution for the true medical prognoses (X_t and Y_t) and doctor errors ($\mathcal{E}_t^X$ and $\mathcal{E}_t^Y$) ensures that the predicted prognoses ($\hat{X}_t$ and $\hat{Y}_t$) fall between the signals and the priors (μ_{Xt} and μ_{Yt}). This property extends to any symmetric and quasiconcave probability density functions for the true medical prognoses and doctor errors (Chambers and Healy 2012). Consequently, we believe that our insights remain valid whenever X_t, Y_t, $\mathcal{E}_t^X$ and $\mathcal{E}_t^Y$ follow symmetric and quasiconcave probability densities.

We now turn to describing our model under PC or CP. Under PC, the doctor shares average prognosis values ($\hat{X}_t = \mu_{Xt}$ and $\hat{Y}_t = \mu_{Yt}$), while under CP, the doctor shares personalized prognosis predictions in (11). The patient's perceived utility $\hat{U}_{it}(.)$ is defined for PC and CP in (8) and (9), respectively. Here, γ_{it} represents type-i patient's random error, assumed to be identically normally distributed with mean zero and variance $\sigma^2_{p,i}/2$, where $\sigma_{p,1}$ and $\sigma_{p,2}$ capture levels of patient irrationality. We assume that γ_{it}'s are independent across treatments and patients, and from X_t's, Y_t's, $\mathcal{E}^X_t$'s, and $\mathcal{E}^Y_t$'s.

The probability that a type-i patient selects treatment A is given by:

$$P_i(w_i, \hat{x}_t, \hat{y}_t) = \mathbb{P}_\gamma\left(\hat{U}_A(w_i, \hat{x}_A, \hat{y}_A) \geq \hat{U}_B(w_i, \hat{x}_B, \hat{y}_B)\right) = \Phi\left(\frac{w_i(\hat{x}_A - \hat{x}_B) + (1-w_i)(\hat{y}_A - \hat{y}_B)}{\sigma_{p,i}}\right), \tag{12}$$

where $\Phi(.)$ is the standard normal cumulative distribution function.

4. How Should Doctors Optimally Account for Cognitive Limitations?

We begin by examining how doctors can address their informational and cognitive limitations—both in assessing patients medically and in their ability to effectively communicate and apply patient preferences into SDM. To do so, it is helpful to define the parameters λ_X and λ_Y from (11), relative to Bayesian benchmarks:

$$\lambda_X = \alpha_d \lambda_X^{bayes}, \qquad \lambda_Y = \alpha_d \lambda_Y^{bayes}, \tag{13}$$

where $\lambda_X^{bayes} = \frac{\sigma^2_X}{\sigma^2_X + \sigma^2_{d,X}}$ and $\lambda_Y^{bayes} = \frac{\sigma^2_Y}{\sigma^2_Y + \sigma^2_{d,Y}}$ represent Bayesian weights on prognosis signals for dimensions x and y, respectively. Here, $\alpha_d = 1$ reflects a doctor using Bayesian weights, while $\alpha_d > 1$ or $\alpha_d < 1$ indicates placing more or less weight on the signals than the Bayesian benchmark.

In models using standardized medical prognoses (SM and PC), doctors do not personalize medical prognoses. Thus, we focus on how doctors should handle random errors when personalizing medical prognoses under PM and CP. Optimal weighting, denoted by α^*_d, maximizes the expected utilities EU^{PM} and EU^{CP}. The following proposition characterizes α^*_d.

Proposition 1 *(a) Under PM, $\alpha^*_d = 1$. (b) Under CP, $\alpha^*_d > 1$ if $|\Delta\mu_1|$ and $|\Delta\mu_2|$ are small; $\alpha^*_d < 1$ if they are large.*

Under PM where doctors subject treatment decisions only to their own errors, Proposition 1(a) shows that doctors should mitigate their own prognosis noise by placing the Bayesian weight ($\alpha_d = 1$) on the priors. However, under CP, Proposition 1(b) shows that doctors should adjust weight depending

on patient preferences. Specifically, they should reduce weight on priors if patients have weak preferences between treatments (small $|\Delta\mu_1|$ and $|\Delta\mu_2|$) and increase it if preferences are strong (large $|\Delta\mu_1|$ and $|\Delta\mu_2|$).

The reasoning behind this is that patients with bounded rationality are more likely to choose the wrong treatment when treatment utilities, based on the medical prognoses provided, are similar. Therefore, prognosis values that create a clear difference between the utilities of the two treatments help reduce the likelihood that patients will make the wrong choice. Thus, the doctor's optimal weighting of priors (μ_{Xt} and μ_{Yt}) and signals depends on whether the treatment utilities derived from the priors are close or far apart. The terms $\Delta\mu_1$ and $\Delta\mu_2$ in (3) and (4) represent the utility differences between treatment A and B for type-1 and type-2 patients, respectively, based on the priors. Thus, (i) when patients have weak preferences (small $|\Delta\mu_1|$ and $|\Delta\mu_2|$), the treatment utilities calculated using the priors are similar, and (ii) when patients have strong preferences (large $|\Delta\mu_1|$ and $|\Delta\mu_2|$), the treatment utilities calculated using the priors are already significantly different. To ensure a significant difference in the utilities of the two treatments, in case (i), it is beneficial to deviate from the priors by placing less weight on them than the Bayesian weight, while in case (ii), it is beneficial to adhere closely to the priors by placing more weight on them than the Bayesian weight.

In the remainder of the paper, we acknowledge that doctors are not able to place the optimal weight on the signals. In Section 5, we will show that neglecting these errors in prognosis-sharing significantly affects the conditions under which prognosis personalization is beneficial.

5. When Should Clinical Guidelines Advise Personalized Prognoses?

In this section, we analyze when clinical guidelines should encourage doctors to personalize medical prognoses. Personalization holds promise because it can improve patient outcomes by accounting for individual treatment responses. However, because doctors' prognosis predictions are inherently noisy and may not fully account for patients' decision errors, personalizing medical prognoses also risks exacerbating the effects of patients' cognitive limitations.

To balance these factors, we assess the net effect of prognosis personalization by focusing on two aspects: doctor accuracy and patient prognosis heterogeneity. To quantify the level of prognosis heterogeneity *relative to* doctor error, we introduce two metrics, r_X and r_Y, defined as follows:

$$r_X := \frac{\sigma_X^4}{\sigma_X^2 + \sigma_{d,X}^2}, \quad r_Y := \frac{\sigma_Y^4}{\sigma_Y^2 + \sigma_{d,Y}^2}. \tag{14}$$

These ratios, which we call "prognosis personalization ratios" for the dimensions x and y, increase in prognosis heterogeneity and decrease in doctor error. In this section, we consider two types of settings: settings without patient participation and settings with patient participation. Table 2 summarizes our key findings, which we discuss in detail in Sections 5.1 and 5.2.

Patient participation	Conditions	Personalize medical prognoses?	Corresponding result
No	Doctors with $\alpha_d = 1$ (Bayesian)	Yes	Prop. 2
	Doctors with $\alpha_d < 1$	Yes	Prop. 3a
	Doctors with $\alpha_d > 1$	Yes if r_X and r_Y high; No if low	Prop. 3b
Yes	Small $\lvert\Delta\mu_1\rvert$ and $\lvert\Delta\mu_2\rvert$	Yes	Prop. 4
	Large $\lvert\Delta\mu_1\rvert$ and $\lvert\Delta\mu_2\rvert$	Yes if r_X and r_Y high; No if low	Prop. 5a and 5b

Table 2 Summary of Section 5 findings.

5.1. Medical Prognoses Personalization without Patient Participation (PM vs SM)

In this subsection, we consider settings in which patients do not participate in decision-making, and so their errors do not affect treatment decisions. Our goal is to determine when PM outperforms SM. This comparison depends on the doctor's weighting of signals relative to the Bayesian benchmark. Therefore, we divide our analysis into two cases: Bayesian and non-Bayesian doctors.

5.1.1. Bayesian Doctors. We start by comparing the expected utilities of PM and SM for Bayesian doctors ($\alpha_d = 1$).

Proposition 2 *If $\alpha_d = 1$, then $EU^{PM} > EU^{SM}$, and EU^{PM} increases with r_X and r_Y.*

Proposition 2 establishes that, in settings without patient participation and with Bayesian doctors, PM outperforms SM. Proposition 2 further shows that when prognosis personalization ratios (r_X and r_Y) are high, i.e., when medical prognosis heterogeneity (σ_X, σ_Y) is high relative to doctor error ($\sigma_{d,X}$, $\sigma_{d,Y}$), it becomes most valuable to advise doctors to switch from SM to PM. The rationale is as follows. As discussed at the beginning of Section 5, switching from SM to PM has two opposing effects: it can enhance patient utility by addressing heterogeneity in patient responses, but it also exposes treatment decisions to human cognitive limitations due to doctors' noisy prognoses predictions. As such, to ensure that PM outperforms SM, it becomes crucial for doctors under PM to adjust their weighting of priors to account for their own noise. Bayesian doctors under PM are able to do so because they place the optimal weight on priors as per Proposition 1(a).

5.1.2. Non-Bayesian Doctors. For non-Bayesian doctors, we explore whether PM still outperforms SM, given that these doctors do not optimally weight signals as Bayesian doctors do.

Proposition 3 *Let doctors be non-Bayesian. Then:*
(a) If $\alpha_d < 1$, then $EU^{PM} > EU^{SM}$, and EU^{PM} increases with r_X and r_Y.
(b) If $\alpha_d > 1$, for a fixed r_Y (r_X), there exists a constant ψ_X (ψ_Y) such that $EU^{PM} > EU^{SM}$ if and only if $r_X \geq \psi_X$ ($r_Y \geq \psi_Y$). Furthermore, EU^{PM} increases with r_X and r_Y if and only if r_X and r_Y are sufficiently large, and it decreases otherwise.

Proposition 3(a) establishes that for doctors with $\alpha_d < 1$, PM continues to yield higher expected utility than SM. The rationale is as follows. As noted earlier, prognosis personalization (i) offers the benefit of tailoring treatments to individual medical outcomes, but (ii) also increases decision errors due to doctors' imprecise predictions. Indeed, the second (negative) effect can dominate the first only if the doctor under PM places more weight on the signal than the optimal weight. However, under PM, doctors with $\alpha_d < 1$ underweight the signals relative to the optimal weight as per Proposition 1(a), thereby preventing the negative effect from becoming dominant. Furthermore, Proposition 3(a) suggests that the value of switching from SM to PM increases with higher prognosis personalization ratios (r_X and r_Y). As these ratios increase—indicating increased prognosis heterogeneity (σ_X and σ_Y) or reduced doctor error ($\sigma_{d,X}$ and $\sigma_{d,Y}$)—the need for prognosis personalization becomes more pronounced.

On the other hand, since under PM, doctors with $\alpha_d > 1$ place excessive weights on the signals relative to the optimal weight, as established by Proposition 1(a), the risk of decision errors may outweigh the benefits, depending on the level of r_X and r_Y. Specifically, Proposition 3(b) establishes that the benefits of personalization outweigh the negative effect of decision errors if and only if the prognosis personalization ratios (r_X and r_Y) are sufficiently low.

Moreover, Proposition 3(b) interestingly shows that for doctors with $\alpha_d > 1$, the expected utility of PM does not necessarily increase in the prognosis personalization ratios (r_X and r_Y). Due to this non-monotone behavior of the expected utility under PM, the largest gain from implementing SM (rather than PM) occurs not at low, but at moderate levels of r_X and r_Y (i.e., when there is moderate medical prognosis heterogeneity relative to doctor error). This is due to two opposing effects: Higher ratios (indicating greater prognosis heterogeneity relative to doctor error) increase the need to tailor treatments to individual medical outcomes. On the other hand, as r_X and r_Y increase—either due to increased heterogeneity or decreased doctor error—, the weight placed on the noisy signal by the doctor with $\alpha_d > 1$ becomes even greater (recall (11) and (13)). Depending on which effect is stronger, the expected utility of PM may increase or decrease in r_X and r_Y. Proposition 3(b) further confirms, as expected, that implementing PM (rather than SM) is most beneficial at high r_X and r_Y since the expected utility of PM increases with r_X and r_Y if r_X and r_Y are sufficiently large.

5.2. Medical Prognoses Personalization with Patient Participation (CP vs PC)

In this subsection, we consider settings in which patients participate in decision-making, and so their errors affect treatment decisions. Here, we ask: When does CP outperform PC?

In contrast to Section 5.1, here, since boundedly rational patients are involved in decision-making, placing the Bayesian weights on the priors in response to human cognitive limitations is no longer

optimal as per Proposition 1(b). Hence, it is not obvious whether prognosis personalization is still beneficial for Bayesian doctors ($\alpha_d = 1$) and those with $\alpha_d < 1$. Indeed, in settings with patient participation, when to encourage doctors to switch from PC to CP depends on the degree of patient preference. We start by comparing CP and PC when patients have weak preferences between treatments.

Proposition 4 *If $|\Delta\mu_1|$ and $|\Delta\mu_2|$ are small, then $EU^{CP} > EU^{PC}$, and EU^{CP} increases with r_X and r_Y.*

Proposition 4 demonstrates that if patients have weak preferences between treatments (small $|\Delta\mu_1|$ and $|\Delta\mu_2|$), CP outperforms PC. Furthermore, as medical prognosis heterogeneity increases relative to doctor errors (i.e., as r_X and r_Y increase), the utility gain from prognosis personalization becomes larger. This effect is observed even for doctors with $\alpha_d > 1$, in contrast to Section 5.1. The reasons for this are as follows. First, as detailed in Section 5.1, to ensure that prognosis personalization increases patient utility, it becomes crucial for doctors not to place too much weight on the signal relative to the optimal weight. However, for boundedly rational patients with weak preferences, doctors with $\alpha_d > 1$ actually apply the optimal weight to the signal, as per Proposition 1(b). Second, when patients have weak preferences between treatments, it becomes less clear a priori which treatment is most appropriate. In these cases, personalizing medical prognoses by providing patient-specific information can significantly reduce this uncertainty and improve patient outcomes.

Next, we compare CP and PC when patients have strong preferences between treatments.

Proposition 5 *If $|\Delta\mu_1|$ and $|\Delta\mu_2|$ are large, then:*
(a) For sufficiently high r_X and r_Y, $EU^{CP} > EU^{PC}$, and EU^{CP} increases with r_X and r_Y.
(b) For sufficiently low r_X and r_Y, $EU^{CP} < EU^{PC}$, and EU^{CP} decreases with r_X and r_Y.
(c) The effect of r_X and r_Y on the optimality of CP or PC may not be monotone.

Proposition 5(a) demonstrates that in settings where patients with strong preferences are involved in decision-making (large $|\Delta\mu_1|$ and $|\Delta\mu_2|$), prognosis personalization increases patient utility when medical prognosis heterogeneity is high relative to doctor errors (i.e., high r_X and r_Y). Furthermore, the utility gain from prognosis personalization tends to become more substantial as r_X and r_Y increase.

On the other hand, Proposition 5(b) establishes that prognosis personalization can lead to a utility loss when medical prognosis heterogeneity is low relative to doctor errors (i.e., r_X and r_Y are low). Proposition 5(b) further confirms that the utility loss from prognosis personalization does not always

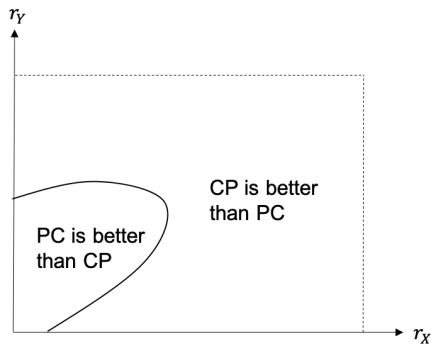

Figure 1 Illustration of Proposition 5(c).

decrease as prognosis heterogeneity becomes larger relative to doctor error (i.e., as r_X and r_Y become larger). These findings hold even for Bayesian doctors and those with $\alpha_d < 1$, contrary to Section 5.1. The intuition is as follows. As discussed earlier, although prognosis personalization has the potential to address the heterogeneous needs of each patient, it can lead to significant utility losses if the doctor overreacts to the noisy signals by placing too much weight on them compared to the optimal weight. When boundedly rational patients with strong preferences are involved in decision-making, Bayesian doctors and even those with $\alpha_d < 1$ "overweight" the signal relative to the optimal weight, as established by Proposition 1(b). Thus, it is possible that prognosis personalization may be detrimental even for Bayesian doctors and those with $\alpha_d < 1$.

Proposition 5(c) shows that the effect of r_X and r_Y on the optimality of prognosis personalization can be non-monotonic. For example, the optimal policy may switch from CP to PC and then from PC to CP as r_X or r_Y increase (e.g., see Figure 1). One of the reasons for this is that the expected utility of CP is not monotone in r_X and r_Y. That is, it decreases in r_X and r_Y if r_X and r_Y are sufficiently low, while it increases in r_X and r_Y if r_X and r_Y are sufficiently high, as established by Proposition 5(a) and Proposition 5(b). Another reason is that in settings with patient participation, an increase in r_X or r_Y affects the utilities of type-1 and type-2 patients differently. This is because after learning their individual prognoses from the doctor, each patient type predicts the utility of each treatment by (i) applying different weights (w_1 and w_2) to the prognoses, and (ii) incorporating random errors with different variances ($\sigma_{p,1}$ and $\sigma_{p,2}$). As a result of this heterogeneous effect of an increase in r_X or r_Y on type-1 and type-2 patients, whether CP or PC is optimal does not follow a straightforward pattern.

6. When Should Clinical Guidelines Advise Patient Participation?

In this section, we analyze when clinical guidelines should recommend patient participation in the decision-making process. Specifically, we address the questions: When does PC outperform SM?, and when does CP outperform PM?

Patient participation can improve patient utility by accounting for different preferences. However, because patients may not fully understand medical prognoses or accurately apply their personal preferences due to bounded rationality, patient participation also introduces potential decision errors. Thus, the net impact of patient participation depends on the degree of patient error ($\sigma_{p,1}$ and $\sigma_{p,2}$) and the degree of variation in treatment responses between patient types. We quantify this response variation with a measure d_p:

$$d_p = \Delta\mu_1 - \Delta\mu_2 = (w_1 - w_2)(\mu_{XA} - \mu_{XB}) + (w_2 - w_1)(\mu_{YA} - \mu_{YB}). \tag{15}$$

A larger d_p indicates stronger preferences for different treatments among patient types. The following proposition identifies the necessary and sufficient conditions under which patient participation (PC or CP) yields greater utility than no patient participation (SM or PM).

Proposition 6 *(a) For given $\sigma_{p,1}$ and $\sigma_{p,2}$, there exists a threshold $f(\sigma_{p,1}, \sigma_{p,2})$ such that $EU^{PC} > EU^{SM}$ if and only if $d_p \geq f(\sigma_{p,1}, \sigma_{p,2})$. The threshold $f(\sigma_{p,1}, \sigma_{p,2})$ increases with $\sigma_{p,1}$ and $\sigma_{p,2}$.*
(b) For given $\sigma_{p,1}$ and $\sigma_{p,2}$, there exists a threshold $g(\sigma_{p,1}, \sigma_{p,2})$ such that $EU^{CP} > EU^{PM}$ if $d_p \geq g(\sigma_{p,1}, \sigma_{p,2})$.[5] The threshold $g(\sigma_{p,1}, \sigma_{p,2})$ increases with $\sigma_{p,1}$ and $\sigma_{p,2}$, provided these values are sufficiently large.

Proposition 6 shows that PC and CP tend to perform better than SM and PM, respectively, as patient error levels ($\sigma_{p,1}$ and $\sigma_{p,2}$) decrease and treatment response differences (d_p) increase. In other words, patient participation is beneficial when d_p is high *relative to* patient error. Higher patient error increases the risk of choosing the wrong treatment, which weakens the performance of PC and CP relative to SM and PM, respectively. Furthermore, recall that the doctor does not know whether the patient is type-1 or type-2. As d_p increases, the potential benefit of reducing uncertainty about patient type by involving patients in decision-making increases, thereby improving patient utility.

7. SDM Optimal Design Decision Tree Summary

In Sections 4, 5, and 6, we analyzed (i) how doctors should manage their cognitive limitations, (ii) when they should personalize medical prognoses, and (iii) when patient participation is beneficial. This section synthesizes these findings to outline the optimal design of the prognosis-to-treatment process, identifying the sufficient conditions under which SM, PM, PC, or CP are superior. Figure 2 presents a summary of these findings in a decision tree, which is explained in detail below.

[5] As long as $\alpha_d \geq 0.5$ holds, this condition is also necessary for $EU^{CP} > EU^{PM}$ to hold.

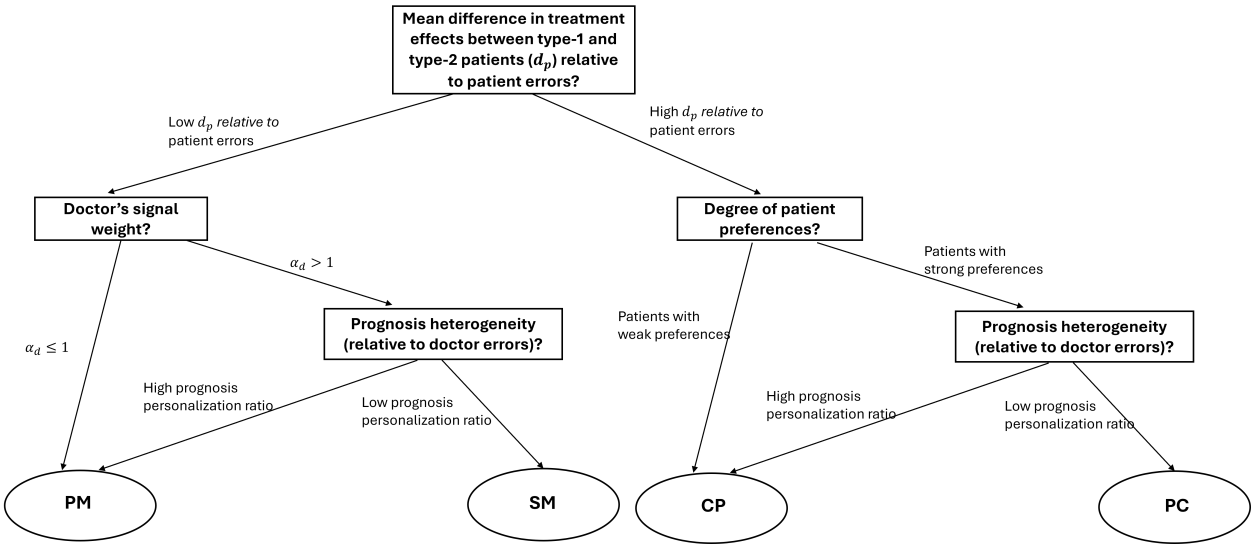

Figure 2 The best design of prognosis-to-treatment processes.

When the mean difference in treatment effects between type-1 and type-2 patients (d_p) is small relative to patient error ($\sigma_{p,1}$ and $\sigma_{p,2}$), Proposition 6 suggests avoiding patient participation (i.e., SM or PM is superior). In settings without patient participation, Propositions 2 and 3 establish that medical prognosis personalization (PM) is beneficial as long as doctors do not over-rely on noisy signals ($\alpha_d \leq 1$). However, if doctors assign more weight to these signals than the Bayesian benchmark ($\alpha_d > 1$), it may be preferable to standardize prognoses (SM) when prognosis personalization ratios (r_X and r_Y) are low.

On the other hand, when the treatment effect difference (d_p) is large relative to patient error ($\sigma_{p,1}$ and $\sigma_{p,2}$), Proposition 6 demonstrates that patient participation is beneficial (i.e., PC or CP is superior). In these settings, Propositions 4 and 5 show that medical prognosis personalization (CP) is advantageous when patients have weak preferences between treatments. However, for patients with strong preferences, it may be better to avoid medical prognosis personalization (PC) when the prognosis personalization ratios (r_X and r_Y) are low.

So far, we have identified when SM, PM, PC, or CP is the optimal policy. However, it is equally important to understand when these policies significantly outperform others. The following corollary provides insight into the conditions under which each approach provides the greatest benefit:

Corollary 1 *(a) SM is most valuable when d_p is low relative to $\sigma_{p,1}$ and $\sigma_{p,2}$, $\alpha_d > 1$, and r_X and r_Y are moderate.*

(b) PM is most valuable when d_p is low relative to $\sigma_{p,1}$ and $\sigma_{p,2}$, $\alpha_d = 1$, and r_X and r_Y are high.

(c) PC is most valuable when d_p is high relative to $\sigma_{p,1}$ and $\sigma_{p,2}$, $\alpha_d > 1$, and r_X and r_Y are moderate.

(d) CP is most valuable when d_p is high relative to $\sigma_{p,1}$ and $\sigma_{p,2}$, $\alpha_d < 1$, and r_X and r_Y are high.

Interestingly, Corollary 1(a) and 1(c) show that policies without medical prognosis personalization (SM or PC) significantly outperform those with medical prognosis personalization (PM or CP) when prognosis heterogeneity, relative to the level of doctor errors, is moderate (i.e., moderate r_X and r_Y values) rather than low. This is because under SM and PC, expected utility remains constant across prognosis personalization ratios r_X and r_Y, whereas under PM and CP, it exhibits a non-monotonic pattern as these ratios vary, as shown in Propositions 3(b), 5(a), and 5(b).

Furthermore, Corollary 1(d) presents a surprising insight: CP performs significantly better than the other policies not with Bayesian doctors, but rather when doctors place less weight on noisy private information than the Bayesian weight. This is because a high d_p implies strong patient preferences (large $|\Delta\mu_1|$ and $|\Delta\mu_2|$, see (15)), and in such settings, placing less weight on noisy signals than the Bayesian weight becomes the optimal weighting policy, as per Proposition 1(b).

8. Conclusions

8.1. Managerial Implications

In medical care, treatment personalization has two key dimensions: medical prognoses and patient preferences. While the existing literature generally supports personalizing both dimensions, this study presents a more nuanced approach. Beyond SM, which does not allow for personalization, and CP, which personalizes both dimensions, we introduce intermediate approaches—PM and PC—that allow for single-dimension personalization, providing valuable guidance on when to avoid, partially implement, or fully adopt personalization.

Our findings suggest that when (i) prognosis heterogeneity is high relative to doctor error and (ii) the treatment effect differences are small relative to patient error, personalizing treatment based on both medical prognoses and patient preferences should be encouraged. However, in contexts where patient or doctor errors predominates, personalizing treatment to account for only patient preferences or only medical prognoses may lead to better outcomes than full personalization, challenging the common view that full personalization is always optimal.

For policymakers who favor full personalization, reducing random errors in patients and doctors is crucial. Training programs to improve patients' health literacy can reduce patient-side errors (Muscat et al. 2019). Alternatively, doctors could elicit patient preferences and integrate them into their decisions, rather than fully shifting decision-making to patients. Additionally, pooling forecasts from multiple doctors—using the "wisdom of the crowd"—can help (Surowiecki 2005, Sunstein 2006, Sjöberg 2009, Davis-Stober et al. 2014), as shown by Kattan et al. (2016), who found that averaging the predictions of as few as five clinicians yielded prediction accuracy comparable to the that of the

best single clinician. Group activities and team-based care, such as case conferences, expert consultations, and morning rounds, are traditional methods to leverage the "wisdom of the crowd" (Radcliffe et al. 2019).

The coproduction framework outlined in this paper has relevance in many other service contexts, ranging from supply chain management to financial planning. For example, in retail distribution, retailers and brand manufacturers work together to allocate store resources. Manufacturers provide demand forecasts that retailers use to make space allocation decisions. Similarly, in financial planning, consultants offer guidance to individual investors, who then make decisions based on that information. We believe that the insights developed here can be applied across such diverse service contexts.

8.2. Limitations and Future Directions

This study has several limitations, which can serve as potential directions for future research. First, our examination of the coproduction process primarily involves the doctor acting as a "technical expert," providing patients with relevant information, and allowing patients to make the final treatment decision. However, coproduction can take other forms, such as doctors incorporating patient preferences directly into treatment choices. Future work could compare these approaches to find the optimal design for coproduction.

Second, we did not account for the time that personalization requires. In practice, personalization typically consumes more of a doctor's time than standardized methods. Future research could explore the time aspects of personalization and its impact on clinical workflows.

Third, our model considers personalizing prognoses along both the x and y dimensions. Exploring optimal designs where only one dimension is personalized could reveal useful insights.

Lastly, we assumed a linear utility model with two treatment options. Future work may delve into scenarios involving a non-linear utility model and offer a broader perspective on personalization.

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APPENDIX

The appendices are organized as follows. In Appendix A, we derive the exact expressions for the expected utilities under SM, PM, PC, and CP. In Appendix B, we present proofs for the results in the present paper.

Throughout the proofs, $\phi(z)$ and $\Phi(z)$ denote the probability density function (pdf) and the cumulative distribution function (cdf) of the standard normal distribution at point z, respectively. $f(x;\mu,\sigma)$ represents normal pdf with mean μ and standard deviation σ at point x. Let $\Delta\mu_X$, $\Delta\mu_Y$, and $\Delta\mu$ denote:

$$\Delta\mu_X = \mu_{XA} - \mu_{XB}, \quad \Delta\mu_Y = \mu_{YA} - \mu_{YB}, \text{ and } \Delta\mu = \bar{w}\Delta\mu_X + (1-\bar{w})\Delta\mu_Y. \tag{16}$$

Recall that we assume $\Delta\mu_X > 0$, $\Delta\mu_Y < 0$, and $\Delta\mu < 0$. Furthermore, let us define $\bar{\sigma}_i$ and $\bar{\sigma}$ as:

$$\bar{\sigma}_i = \sqrt{w_i^2 r_X + (1-w_i)^2 r_Y} \text{ for } i = 1,2, \text{ and } \bar{\sigma} = \sqrt{\bar{w}^2 r_X + (1-\bar{w})^2 r_Y}, \tag{17}$$

where $\bar{w}$, r_X, and r_Y are given with (2), and (14), respectively.

Throughout the appendix, EU^{SM}, EU^{PC}, EU^{PM}, and EU^{CP} represent the expected utility of SM, PC, PM, and CP, respectively. Finally, $\mathcal{I}\{A\}$ denotes the indicator random variable associated with event A that has value 1 if event A occurs and has value 0 otherwise.

Appendix A: Derivation of the Expected Utilities under SM, PM, PC, and CP

Lemma A1 *The expected utilities of SM, PC, PM, and CP, denoted by EU^{SM}, EU^{PC}, EU^{PM}, and EU^{CP}, respectively, are equal to:*

$$EU^{SM} = \bar{w}\mu_{XB} + (1-\bar{w})\mu_{YB}, \tag{18}$$

$$EU^{PM} = \Delta\mu\Phi\left(\frac{\Delta\mu}{\alpha_d\bar{\sigma}}\right) + \bar{\sigma}\phi\left(\frac{\Delta\mu}{\alpha_d\bar{\sigma}}\right) + \bar{w}\mu_{XB} + (1-\bar{w})\mu_{YB}, \tag{19}$$

$$EU^{PC} = pEU_1^{PC} + (1-p)EU_2^{PC} + \bar{w}\mu_{XB} + (1-\bar{w})\mu_{YB}, \tag{20}$$

$$EU^{CP} = pEU_1^{CP} + (1-p)EU_2^{CP} + \bar{w}\mu_{XB} + (1-\bar{w})\mu_{YB}. \tag{21}$$

In (20) and (21), EU_i^{PC} and EU_i^{CP} are equal to:

$$EU_i^{PC} = \Delta\mu_i\Phi\left(\frac{\Delta\mu_i}{\sigma_{p,i}}\right), \tag{22}$$

$$EU_i^{CP} = \Delta\mu_i\Phi\left(\frac{\Delta\mu_i}{\sqrt{\alpha_d^2\bar{\sigma}_i^2 + \sigma_{p,i}^2}}\right) + \frac{\alpha_d\bar{\sigma}_i^2}{\sqrt{\alpha_d^2\bar{\sigma}_i^2 + \sigma_{p,i}^2}}\phi\left(\frac{\Delta\mu_i}{\sqrt{\alpha_d^2\bar{\sigma}_i^2 + \sigma_{p,i}^2}}\right), \tag{23}$$

where $\Delta\mu_1$, $\Delta\mu_2$, $\Delta\mu$, $\bar{\sigma}_i$ and $\bar{\sigma}$ are given with (16) and (17), respectively, and $\Phi(.)$ and $\phi(.)$ are cdf and pdf of the standard normal distribution, respectively.

Proof: *Part I:* Since under SM, doctors always choose treatment B, the expected utility of SM is equal to:

$$\begin{aligned} EU^{SM} &= p\mathbb{E}_{X_B,Y_B}\left[w_1X_B + (1-w_1)Y_B\right] + (1-p)\mathbb{E}_{X_B,Y_B}\left[w_2X_B + (1-w_2)Y_B\right] \\ &= \bar{w}\mu_{XB} + (1-\bar{w})\mu_{YB}, \end{aligned} \tag{24}$$

where the subscripts in the expectations indicate what variable the expectation is taken over. The first term in the first equality corresponds to the event that the doctor sees a type-1 patient, whereas the second term corresponds to the event

that the doctor sees a type-2 patient. Furthermore, recalling that μ_{XB} and μ_{YB} denote the expectations of X_B and Y_B, respectively, the second equality follows from (2).

Part II: The expected utility of PM is equal to:

$$
\begin{aligned}
&EU^{PM} \\
&= p\mathbb{E}_{S_t^X,S_t^Y,X_t,Y_t}\Big[(w_1X_A+(1-w_1)Y_A)\mathcal{I}\{\bar{w}(\hat{X}_A-\hat{X}_B)+(1-\bar{w})(\hat{Y}_A-\hat{Y}_B)\geq 0\} && (25)\\
&\qquad +(w_1X_B+(1-w_1)Y_B)\mathcal{I}\{\bar{w}(\hat{X}_A-\hat{X}_B)+(1-\bar{w})(\hat{Y}_A-\hat{Y}_B)<0\}\Big] && (26)\\
&+(1-p)\mathbb{E}_{X_t,Y_t,S_t^X,S_t^Y}\Big[(w_2X_A+(1-w_2)Y_A)\mathcal{I}\{\bar{w}(\hat{X}_A-\hat{X}_B)+(1-\bar{w})(\hat{Y}_A-\hat{Y}_B)\geq 0\} && (27)\\
&\qquad +(w_2X_B+(1-w_2)Y_B)\mathcal{I}\{\bar{w}(\hat{X}_A-\hat{X}_B)+(1-\bar{w})(\hat{Y}_A-\hat{Y}_B)<0\}\Big] && (28)\\
&= \mathbb{E}_{S_t^X,S_t^Y,X_t,Y_t}\Big[(\bar{w}X_A+(1-\bar{w})Y_A)\mathcal{I}\{\bar{w}(\hat{X}_A-\hat{X}_B)+(1-\bar{w})(\hat{Y}_A-\hat{Y}_B)\geq 0\} \\
&\qquad +(\bar{w}X_B+(1-\bar{w})Y_B)\mathcal{I}\{\bar{w}(\hat{X}_A-\hat{X}_B)+(1-\bar{w})(\hat{Y}_A-\hat{Y}_B)<0\}\Big], && (29)
\end{aligned}
$$

where the term in (25) corresponds to the event that the doctor sees a type-1 patient and treatment A is selected, the term in (26) corresponds to the event that the doctor sees a type-1 patient and treatment B is selected, whereas term in (27) corresponds to the event that the doctor sees a type-2 patient and treatment A is selected, and term in (28) corresponds to the event that the doctor sees a type-2 patient and treatment B is selected. Furthermore, the second equality follows from the linearity of expectation and from (2). For algebraic convenience, we will first derive the expression for the expected utility difference between PM and SM by combining (18) and (29), and then find the exact expression for EU^{PM}:

$$
\begin{aligned}
&EU^{PM}-EU^{SM} \\
&= \mathbb{E}_{S_t^X,S_t^Y,X_t,Y_t}\left[(\bar{w}(X_A-X_B)+(1-\bar{w})(Y_A-Y_B))\mathcal{I}\left\{\bar{w}(\hat{X}_A-\hat{X}_B)+(1-\bar{w})(\hat{Y}_A-\hat{Y}_B)\geq 0\right\}\right] \\
&= \bar{w}\mathbb{E}_{S_t^X,S_t^Y,X_t}\left[(X_A-X_B)\mathcal{I}\left\{\bar{w}(\hat{X}_A-\hat{X}_B)+(1-\bar{w})(\hat{Y}_A-\hat{Y}_B)\geq 0\right\}\right] \\
&\quad +(1-\bar{w})\mathbb{E}_{S_t^X,S_t^Y,Y_t}\left[(Y_A-Y_B)\mathcal{I}\left\{\bar{w}(\hat{X}_A-\hat{X}_B)+(1-\bar{w})(\hat{Y}_A-\hat{Y}_B)\geq 0\right\}\right] \\
&= \bar{w}\mathbb{E}_{S_t^X,S_t^Y}\left[\mathbb{E}_{X_t}\left[(X_A-X_B)\mathcal{I}\left\{\bar{w}(\hat{X}_A-\hat{X}_B)+(1-\bar{w})(\hat{Y}_A-\hat{Y}_B)\geq 0\right\}\middle| S_t^X,S_t^Y\right]\right] \\
&\quad +(1-\bar{w})\mathbb{E}_{S_t^X,S_t^Y}\left[\mathbb{E}_{Y_t}\left[(Y_A-Y_B)\mathcal{I}\left\{\bar{w}(\hat{X}_A-\hat{X}_B)+(1-\bar{w})(\hat{Y}_A-\hat{Y}_B)\geq 0\right\}\middle| S_t^X,S_t^Y\right]\right] \\
&= \bar{w}\mathbb{E}_{S_t^X,S_t^Y}\left[\mathbb{E}_{X_t}\left[(X_A-X_B)\mid S_t^X,S_t^Y\right]\mathcal{I}\left\{\bar{w}(\hat{X}_A-\hat{X}_B)+(1-\bar{w})(\hat{Y}_A-\hat{Y}_B)\geq 0\right\}\right] \\
&\quad +(1-\bar{w})\mathbb{E}_{S_t^X,S_t^Y}\left[\mathbb{E}_{Y_t}\left[(Y_A-Y_B)\mid S_t^X,S_t^Y\right]\mathcal{I}\left\{\bar{w}(\hat{X}_A-\hat{X}_B)+(1-\bar{w})(\hat{Y}_A-\hat{Y}_B)\geq 0\right\}\right] \\
&= \bar{w}\mathbb{E}_{S_t^X,S_t^Y}\left[\left(\mathbb{E}[X_A\mid S_A^X]-\mathbb{E}[X_B\mid S_B^X]\right)\mathcal{I}\left\{\bar{w}(\hat{X}_A-\hat{X}_B)+(1-\bar{w})(\hat{Y}_A-\hat{Y}_B)\geq 0\right\}\right] \\
&\quad +(1-\bar{w})\mathbb{E}_{S_t^X,S_t^Y}\left[\left(\mathbb{E}[Y_A\mid S_A^Y]-\mathbb{E}[Y_B\mid S_B^Y]\right)\mathcal{I}\left\{\bar{w}(\hat{X}_A-\hat{X}_B)+(1-\bar{w})(\hat{Y}_A-\hat{Y}_B)\geq 0\right\}\right], && (30)
\end{aligned}
$$

where the second equality follows from linearity of the expectation, the third equality follows from the law of iterated expectations, the forth equality follows from the fact that when S_t^X and S_t^Y are given, $\mathcal{I}\left\{\bar{w}(\hat{X}_A-\hat{X}_B)+(1-\bar{w})(\hat{Y}_A-\hat{Y}_B)\geq 0\right\}$ is not random anymore, and the fifth equality follows from the independence of S_t^X and S_t^Y. In (30), using (10), (11) and (13), we can write $\hat{X}_A-\hat{X}_B$ and $\hat{Y}_A-\hat{Y}_B$ as:

$$
\hat{X}_A-\hat{X}_B=\alpha_d\frac{\sigma_X^2}{\sigma_X^2+\sigma_{d,X}^2}(S_{kA}^X-S_{kB}^X)+\left(1-\alpha_d\frac{\sigma_X^2}{\sigma_X^2+\sigma_{d,X}^2}\right)\Delta\mu_X=\Delta\mu_X+\alpha_d\sqrt{r_X}Z_X,
$$

$$\hat{Y}_A - \hat{Y}_B = \alpha_d \frac{\sigma_Y^2}{\sigma_Y^2 + \sigma_{d,Y}^2}(S_{kA}^Y - S_{kB}^Y) + \left(1 - \alpha_d \frac{\sigma_Y^2}{\sigma_Y^2 + \sigma_{d,Y}^2}\right) \Delta\mu_Y = \Delta\mu_Y + \alpha_d \sqrt{r_Y} Z_Y, \tag{31}$$

where r_X and r_Y are given with (14), and $\Delta\mu_X$ and $\Delta\mu_Y$ are defined in (16), and Z_X and Z_Y are standard normal random variables. Similarly, $\mathbb{E}[X_A \mid S_A^X] - \mathbb{E}[X_B \mid S_B^X]$ and $\mathbb{E}[Y_A \mid S_A^Y] - \mathbb{E}[Y_B \mid S_B^Y]$ are equal to:

$$\begin{aligned}
\mathbb{E}[X_A \mid S_A^X] - \mathbb{E}[X_B \mid S_B^X] &= \frac{\sigma_X^2}{\sigma_X^2 + \sigma_{d,X}^2}(S_{kA}^X - S_{kB}^X) + \left(1 - \frac{\sigma_X^2}{\sigma_X^2 + \sigma_{d,X}^2}\right) \Delta\mu_X = \Delta\mu_X + \sqrt{r_X} Z_X, \\
\mathbb{E}[Y_A \mid S_A^Y] - \mathbb{E}[Y_B \mid S_B^Y] &= \frac{\sigma_Y^2}{\sigma_Y^2 + \sigma_{d,Y}^2}(S_{kA}^Y - S_{kB}^Y) + \left(1 - \frac{\sigma_Y^2}{\sigma_Y^2 + \sigma_{d,Y}^2}\right) \Delta\mu_Y = \Delta\mu_Y + \sqrt{r_Y} Z_Y.
\end{aligned} \tag{32}$$

Thus, $EU^{PM} - EU^{SM}$ in (30) could be rewritten as:

$$\begin{aligned}
&EU^{PM} - EU^{SM} \\
&= \bar{w}\mathbb{E}_{Z_X, Z_Y}\left[(\Delta\mu_X + \sqrt{r_X} Z_X)\, \mathcal{I}\left\{\bar{w}(\Delta\mu_X + \alpha_d\sqrt{r_X} Z_X) + (1-\bar{w})(\Delta\mu_Y + \alpha_d\sqrt{r_Y} Z_Y) \geq 0\right\}\right] \\
&+ (1-\bar{w})\mathbb{E}_{Z_X, Z_Y}\left[(\Delta\mu_Y + \sqrt{r_Y} Z_Y)\, \mathcal{I}\left\{\bar{w}(\Delta\mu_X + \alpha_d\sqrt{r_X} Z_X) + (1-\bar{w})(\Delta\mu_Y + \alpha_d\sqrt{r_Y} Z_Y) \geq 0\right\}\right] \\
&= \mathbb{E}_{Z_X, Z_Y}\left[(\Delta\mu + \bar{w}\sqrt{r_X} Z_X + (1-\bar{w})\sqrt{r_Y} Z_Y)\, \mathcal{I}\left\{\Delta\mu + \alpha_d(\bar{w}\sqrt{r_X} Z_X + (1-\bar{w})\sqrt{r_Y} Z_Y) \geq 0\right\}\right] \\
&= \mathbb{E}_Z\left[(\Delta\mu + \bar{\sigma} Z)\, \mathcal{I}\left\{\Delta\mu + \alpha_d \bar{\sigma} Z \geq 0\right\}\right] \\
&= \int_{-\frac{\Delta\mu}{\alpha_d \bar{\sigma}}}^{\infty} (\Delta\mu + \bar{\sigma} z)\phi(z) dz \\
&= \Delta\mu \Phi\left(\frac{\Delta\mu}{\alpha_d \bar{\sigma}}\right) + \bar{\sigma}\phi\left(\frac{\Delta\mu}{\alpha_d \bar{\sigma}}\right),
\end{aligned} \tag{33}$$

where $\Delta\mu$ and $\bar{\sigma}$ are given with (16) and (17), respectively, Z is a standard normal random variable, and $\phi(z)$ is the pdf of the standard normal distribution at point z. The first equality follows from substituting the expressions in (31) and (32) for $\hat{X}_A - \hat{X}_B$, $\hat{Y}_A - \hat{Y}_B$, $\mathbb{E}[X_A \mid S_A^X] - \mathbb{E}[X_B \mid S_B^X]$, and $\mathbb{E}[Y_A \mid S_A^Y] - \mathbb{E}[Y_B \mid S_B^Y]$ in (30), the second equality follows from linearity of the expectation and (16), the third equality follows from plugging $\bar{\sigma} Z$ for $\bar{w}\sqrt{r_X} Z_X + (1-\bar{w})\sqrt{r_Y} Z_Y$, which are equal in distribution, the forth equality follows from the definition of the expectation, and the fifth equality follows from algebra. Finally, combining (18) and (33), we conclude that the expected utility of PM is given with (19).

Part III: The expected utility of PC is equal to:

$$\begin{aligned}
EU^{PC} &= p\mathbb{E}_{X_t, Y_t}\Big[(w_1 X_A + (1-w_1)Y_A)\, P_1(w_1, \mu_{Xt}, \mu_{Yt}) + (w_1 X_B + (1-w_1)Y_B)\,(1 - P_1(w_1, \mu_{Xt}, \mu_{Yt}))\Big] \\
&+ (1-p)\mathbb{E}_{X_t, Y_t}\Big[(w_2 X_A + (1-w_2)Y_A)\, P_2(w_2, \mu_{Xt}, \mu_{Yt}) + (w_2 X_B + (1-w_2)Y_B)\,(1 - P_2(w_2, \mu_{Xt}, \mu_{Yt}))\Big] \\
&= p\Big((w_1\mu_{XA} + (1-w_1)\mu_{YA})\, P_1(w_1, \mu_{Xt}, \mu_{Yt}) + (w_1\mu_{XB} + (1-w_1)\mu_{YB})\,(1 - P_1(w_1, \mu_{Xt}, \mu_{Yt}))\Big) \\
&\quad + (1-p)\mathbb{E}_{X_t, Y_t}\Big[(w_2\mu_{XA} + (1-w_2)\mu_{YA})\, P_2(w_2, \mu_{Xt}, \mu_{Yt}) \\
&\qquad\qquad + (w_2\mu_{XB} + (1-w_2)\mu_{YB})\,(1 - P_2(w_2, \mu_{Xt}, \mu_{Yt}))\Big] \\
&= p\Big(w_1(\mu_{XA} - \mu_{XB}) + (1-w_1)(\mu_{YA} - \mu_{YB})\Big) P_1(w_1, \mu_{Xt}, \mu_{Yt}) \\
&\quad + (1-p)\Big(w_2(\mu_{XA} - \mu_{XB}) + (1-w_2)(\mu_{YA} - \mu_{YB})\Big) P_2(w_2, \mu_{Xt}, \mu_{Yt}) \\
&\quad + \bar{w}\mu_{XB} + (1-\bar{w})\mu_{YB} \\
&= p\Delta\mu_1 \Phi\left(\frac{\Delta\mu_1}{\sigma_{p,1}}\right) + (1-p)\Delta\mu_2 \Phi\left(\frac{\Delta\mu_2}{\sigma_{p,2}}\right) + \bar{w}\mu_{XB} + (1-\bar{w})\mu_{YB},
\end{aligned} \tag{34}$$

where $P_i(w_i, \hat{x}_t, \hat{y}_t)$ is given with (12). where the second equality follows from (16), the third equality follows from algebraic manipulations, and the last equality follows from (3), and (4), and substituting the expression in (12) for $P_i(w_i, \hat{x}_t, \hat{y}_t)$.

Part IV: The expected utility of CP is equal to:

$$\begin{aligned} EU^{CP} = p\mathbb{E}_{S_t^X, S_t^Y, X_t, Y_t}\Big[&(w_1 X_A + (1-w_1)Y_A)\, P_1(w_1, \hat{X}_t, \hat{Y}_t) \\ &+ (w_1 X_B + (1-w_1)Y_B)\left(1 - P_1(w_1, \hat{X}_t, \hat{Y}_t)\right)\Big] \\ + (1-p)\mathbb{E}_{S_t^X, S_t^Y, X_t, Y_t}\Big[&(w_2 X_A + (1-w_2)Y_A)\, P_2(w_2, \hat{X}_t, \hat{Y}_t) \\ &+ (w_2 X_B + (1-w_2)Y_B)\left(1 - P_2(w_2, \hat{X}_t, \hat{Y}_t)\right)\Big]. \end{aligned} \tag{35}$$

For algebraic convenience, we will first derive the expression for the expected utility difference between CP and SM by combining (18) and (35), and then find the exact expression for EU^{CP}:

$$\begin{aligned} &EU^{CP} - EU^{SM} \\ &= p\mathbb{E}_{S_t^X, S_t^Y, X_t, Y_t}\left[(w_1(X_A - X_B) + (1-w_1)(Y_A - Y_B))\, P_1(w_1, \hat{X}_t, \hat{Y}_t)\right] \\ &\quad + (1-p)\mathbb{E}_{S_t^X, S_t^Y, X_t, Y_t}\left[(w_2(X_A - X_B) + (1-w_2)(Y_A - Y_B))\, P_2(w_2, \hat{X}_t, \hat{Y}_t)\right] \\ &= p\mathbb{E}_{S_t^X, S_t^Y, X_t, Y_t}\left[(w_1(X_A - X_B) + (1-w_1)(Y_A - Y_B))\, \Phi\left(\frac{w_1(\hat{X}_A - \hat{X}_B) + (1-w_1)(\hat{Y}_A - \hat{Y}_B)}{\sigma_{p,1}}\right)\right] \\ &+ (1-p)\mathbb{E}_{S_t^X, S_t^Y, X_t, Y_t}\left[(w_2(X_A - X_B) + (1-w_2)(Y_A - Y_B))\, \Phi\left(\frac{w_2(\hat{X}_A - \hat{X}_B) + (1-w_2)(\hat{Y}_A - \hat{Y}_B)}{\sigma_{p,2}}\right)\right], \end{aligned}$$

where the second equality follows from plugging the expression in (12) for $P_i(w_i, \hat{X}_t, \hat{Y}_t)$. Following the same steps as in (30) and (33) (i.e., applying the law of iterated expectations, plugging the expressions in (31) and (32) for $\hat{X}_A - \hat{X}_B$, $\hat{Y}_A - \hat{Y}_B$, $\mathbb{E}[X_A \mid S_A^X] - \mathbb{E}[X_B \mid S_B^X]$, and $\mathbb{E}[Y_A \mid S_A^Y] - \mathbb{E}[Y_B \mid S_B^Y]$), we can write $EU^{CP} - EU^{SM}$ as:

$$EU^{CP} - EU^{SM} = p\mathbb{E}_Z\left[(\Delta\mu_1 + \bar{\sigma}_1 Z)\, \Phi\left(\frac{\Delta\mu_1 + \alpha_d\bar{\sigma}_1 Z}{\sigma_{p,1}}\right)\right] + (1-p)\mathbb{E}_Z\left[(\Delta\mu_2 + \bar{\sigma}_2 Z)\, \Phi\left(\frac{\Delta\mu_2 + \alpha_d\bar{\sigma}_2 Z}{\sigma_{p,2}}\right)\right]$$

Rearranging above expression, we have:

$$\begin{aligned} &EU^{CP} - EU^{SM} \\ &= p\int_{-\infty}^{\infty}(\Delta\mu_1 + \bar{\sigma}_1 z)\, \Phi\left(\frac{\Delta\mu_1 + \alpha_d\bar{\sigma}_1 z}{\sigma_{p,1}}\right)\phi(z)dz + (1-p)\int_{-\infty}^{\infty}(\Delta\mu_2 + \bar{\sigma}_2 z)\, \Phi\left(\frac{\Delta\mu_2 + \alpha_d\bar{\sigma}_2 z}{\sigma_{p,2}}\right)\phi(z)dz \\ &= p\left(\Delta\mu_1\Phi\left(\frac{\Delta\mu_1}{\sqrt{\alpha_d^2\bar{\sigma}_1^2 + \sigma_{p,1}^2}}\right) + \frac{\alpha_d\bar{\sigma}_1^2}{\sqrt{\alpha_d^2\bar{\sigma}_1^2 + \sigma_{p,1}^2}}\phi\left(\frac{\Delta\mu_1}{\sqrt{\alpha_d^2\bar{\sigma}_1^2 + \sigma_{p,1}^2}}\right)\right) \\ &\quad + (1-p)\left(\Delta\mu_2\Phi\left(\frac{\Delta\mu_2}{\sqrt{\alpha_d^2\bar{\sigma}_2^2 + \sigma_{p,2}^2}}\right) + \frac{\alpha_d\bar{\sigma}_2^2}{\sqrt{\alpha_d^2\bar{\sigma}_2^2 + \sigma_{p,2}^2}}\phi\left(\frac{\Delta\mu_2}{\sqrt{\alpha_d^2\bar{\sigma}_2^2 + \sigma_{p,2}^2}}\right)\right), \end{aligned} \tag{36}$$

where the first equality follows from the definition of expectation and the last equality follows from algebra. Finally, combining (18) and (36), we conclude that the expected utility of PM is given with (21). ■

Appendix B: Proofs of Results

B.1. Proof of Section 4 Results

For the proof of Proposition 1, we first need the following lemma to establish the structure of EU_i^{CP} given with (23) with respect to α_d.

Lemma A2 *(a)* EU_i^{CP} *given with* (23) *is a quasiconcave function of* α_d *in* $\left[1/2, \min\left\{\frac{\sigma_X^2+\sigma_{d,X}^2}{\sigma_X^2}, \frac{\sigma_Y^2+\sigma_{d,Y}^2}{\sigma_Y^2}\right\}\right]$.
(b) If $|\Delta\mu_i| < \sqrt{\bar{\sigma}_i^2+\sigma_{p,i}^2}$, EU_i^{CP} *increases in* α_d *if* $\alpha_d \leq 1$.
(c) If $|\Delta\mu_i| > \sqrt{\bar{\sigma}_i^2+\sigma_{p,i}^2}$, EU_i^{CP} *decreases in* α_d *if* $\alpha_d \geq 1$.

Proof of Lemma A2: (a) The derivative of EU_i^{CP} in (23) with respect to α_d is:

$$\frac{\partial EU_i^{CP}}{\partial \alpha_d} = \frac{\bar{\sigma}_i^2}{\left(\alpha_d^2\bar{\sigma}_i^2+\sigma_{p,i}^2\right)^{3/2}}\left[\Delta\mu_i^2\left(\frac{\alpha_d^2\bar{\sigma}_i^2}{\alpha_d^2\bar{\sigma}_i^2+\sigma_{p,i}^2}-\alpha_d\right)+\sigma_{p,i}^2\right]\phi\left(\frac{\Delta\mu_i}{\sqrt{\alpha_d^2\bar{\sigma}_i^2+\sigma_{p,i}^2}}\right). \tag{37}$$

Let us represent $G_1(\bar{\sigma}_i, \alpha_d)$ by:

$$G_1(\bar{\sigma}_i, \alpha_d) = \Delta\mu_i^2\left(\theta(\bar{\sigma}_i, \alpha_d) - \alpha_d\right) + \sigma_{p,i}^2, \tag{38}$$

where $\theta(\bar{\sigma}_i, \alpha_d) = \frac{\alpha_d^2\bar{\sigma}_i^2}{\alpha_d^2\bar{\sigma}_i^2+\sigma_{p,i}^2}$ and for notational brevity, we suppress the dependence of $G_1(\bar{\sigma}_i, \alpha_d)$ and $\theta(\bar{\sigma}_i, \alpha_d)$ on $\sigma_{p,i}$. The derivative of $G_1(\bar{\sigma}_i, \alpha_d)$ with respect to α_d is:

$$\frac{\partial G_1(\bar{\sigma}_i, \alpha_d)}{\partial \alpha_d} = \Delta\mu_i^2\left(\frac{2}{\alpha_d}\theta(\bar{\sigma}_i, \alpha_d)(1-\theta(\bar{\sigma}_i, \alpha_d)) - 1\right) \tag{39}$$

$$\leq \Delta\mu_i^2\left(\frac{2}{\alpha_d}\frac{1}{2}\left(1-\frac{1}{2}\right)-1\right) = \Delta\mu_i^2\left(\frac{1}{2\alpha_d}-1\right), \tag{40}$$

where the first inequality follows since $\theta(\bar{\sigma}_i, \alpha_d)(1-\theta(\bar{\sigma}_i, \alpha_d))$ is concave in $\theta(\bar{\sigma}_i, \alpha_d)$ and reaches its maximum value at $\theta(\bar{\sigma}_i, \alpha_d) = 1/2$. Furthermore, we have $\Delta\mu_i^2\left(\frac{1}{2\alpha_d}-1\right) \leq 0$ for all $\alpha_d \geq 1/2$. Hence, $\frac{\partial G_1(\bar{\sigma}_i, \alpha_d)}{\partial \alpha_d} \leq 0$ follows for all $\alpha_d \in \left[1/2, \min\left\{\frac{\sigma_X^2+\sigma_{d,X}^2}{\sigma_X^2}, \frac{\sigma_Y^2+\sigma_{d,Y}^2}{\sigma_Y^2}\right\}\right]$, which means that $G_1(\bar{\sigma}_i, \alpha_d)$ decreases in α_d when $\alpha_d \in \left[1/2, \min\left\{\frac{\sigma_X^2+\sigma_{d,X}^2}{\sigma_X^2}, \frac{\sigma_Y^2+\sigma_{d,Y}^2}{\sigma_Y^2}\right\}\right]$. This confirms that EU_i^{CP} is quasiconcave in α_d when $\alpha_d \in \left[1/2, \min\left\{\frac{\sigma_X^2+\sigma_{d,X}^2}{\sigma_X^2}, \frac{\sigma_Y^2+\sigma_{d,Y}^2}{\sigma_Y^2}\right\}\right]$.

(b) The proof is composed of two parts: In part I, we show that EU_i^{CP} increases in α_d for $\alpha_d \in [1/2, 1]$, whereas in part II, we show that EU_i^{CP} increases in α_d for $\alpha_d \in [0, 1/2]$.

Part I: Setting α_d equal to 1 in (37), the derivative of EU_i^{CP} at $\alpha_d = 1$ is:

$$\left[\frac{\partial EU_i^{CP}}{\partial \alpha_d}\right]_{\alpha_d=1} = \frac{\bar{\sigma}_i^2\sigma_{p,i}^2}{\left(\bar{\sigma}_i^2+\sigma_{p,i}^2\right)^{3/2}}\left[\frac{-\Delta\mu_i^2}{\bar{\sigma}_i^2+\sigma_{p,i}^2}+1\right]\phi\left(\frac{\Delta\mu_i}{\sqrt{\bar{\sigma}_i^2+\sigma_{p,i}^2}}\right) > 0, \tag{41}$$

where the inequality follows from $|\Delta\mu_i| < \sqrt{\bar{\sigma}_i^2+\sigma_{p,i}^2}$. Since the derivative of EU_i^{CP} at $\alpha_d = 1$ is positive by (41) and EU_i^{CP} is a quasiconcave function of α_d for $\alpha_d \in \left[1/2, \min\left\{\frac{\sigma_X^2+\sigma_{d,X}^2}{\sigma_X^2}, \frac{\sigma_Y^2+\sigma_{d,Y}^2}{\sigma_Y^2}\right\}\right]$ by part (a) of this lemma, we have:

$$\frac{\partial EU_i^{CP}}{\partial \alpha_d} > 0 \Leftrightarrow G_1(\bar{\sigma}_i, \alpha_d) > 0 \quad \text{for all } \alpha_d \in [1/2, 1]. \tag{42}$$

From (42), it directly follows that EU_i^{CP} increases in α_d if $\alpha_d \in [1/2, 1]$.

Part II: Recalling (37) and (38) from part (a), we need to show that for $\alpha_d \in [0, 1/2]$, $\frac{\partial EU_i^{CP}}{\partial \alpha_d} > 0 \Leftrightarrow G_1(\bar{\sigma}_i, \alpha_d) > 0$.

Noting that $\frac{\partial G_1(\bar{\sigma}_i, \alpha_d)}{\partial \alpha_d}$ is given with (39), $\frac{\partial^2 G_1(\bar{\sigma}_i, \alpha_d)}{\partial \alpha_d^2}$ is equal to:

$$\frac{\partial^2 G_1(\bar{\sigma}_i, \alpha_d)}{\partial \alpha_d^2} = \Delta\mu_i^2\frac{2\theta(\bar{\sigma}_i, \alpha_d)(1-\theta(\bar{\sigma}_i, \alpha_d))}{\alpha_d^2}(-4\theta(\bar{\sigma}_i, \alpha_d)+1). \tag{43}$$

By (43), $\frac{\partial^2 G_1(\bar{\sigma}_i, \alpha_d)}{\partial \alpha_d^2} > 0$ for all $\theta(\bar{\sigma}_i, \alpha_d) < \frac{1}{4} \Leftrightarrow \alpha_d < \frac{\sigma_{p,i}}{\sqrt{3}\bar{\sigma}_i}$, and $\frac{\partial^2 G_1(\bar{\sigma}_i, \alpha_d)}{\partial \alpha_d^2} < 0$ for all $\theta(\bar{\sigma}_i, \alpha_d) > \frac{1}{4} \Leftrightarrow \alpha_d > \frac{\sigma_{p,i}}{\sqrt{3}\bar{\sigma}_i}$. We have two cases:

Case 1: $\frac{\sigma_{p,i}}{\sqrt{3}\bar{\sigma}_i} \geq \frac{1}{2}$. We have $\frac{\partial^2 G_1(\bar{\sigma}_i,\alpha_d)}{\partial \alpha_d^2} > 0$ for all $\alpha_d \in [0,1/2]$, which implies that $\frac{\partial G_1(\bar{\sigma}_i,\alpha_d)}{\partial \alpha_d}$ increases in α_d for $\alpha_d \in [0,1/2]$. Since $\frac{\partial G_1(\bar{\sigma}_i,\alpha_d)}{\partial \alpha_d} \leq 0$ at $\alpha_d = 1/2$ by (39) and (40), we have $\frac{\partial G_1(\bar{\sigma}_i,\alpha_d)}{\partial \alpha_d} \leq 0$ for all $\alpha_d \in [0,1/2]$, which means that $G_1(\bar{\sigma}_i,\alpha_d)$ decreases in α_d for $\alpha_d \in [0,1/2]$. Since $G_1(\bar{\sigma}_i,\alpha_d) > 0$ at $\alpha_d = 1/2$ by (42), we conclude that $G_1(\bar{\sigma}_i,\alpha_d) > 0$ for all $\alpha_d \in [0,1/2]$.

Case 2: $\frac{\sigma_{p,i}}{\sqrt{3}\bar{\sigma}_i} < \frac{1}{2}$. In the interval $[0,1/2]$, $\frac{\partial G_1(\bar{\sigma}_i,\alpha_d)}{\partial \alpha_d}$ is a unimodal function of α_d, which reaches its maximum at $\alpha_d = \frac{\sigma_{p,i}}{\sqrt{3}\bar{\sigma}_i}$. Thus, the maximum value of $\frac{\partial G_1(\bar{\sigma}_i,\alpha_d)}{\partial \alpha_d}$ in the interval $[0,1/2]$ is

$$\left[\frac{\partial G_1(\bar{\sigma}_i,\alpha_d)}{\partial \alpha_d}\right]_{\alpha_d=\frac{\sigma_{p,i}}{\sqrt{3}\bar{\sigma}_i}} = \Delta\mu_i^2\left(\frac{3\sqrt{3}\bar{\sigma}_i}{8\sigma_{p,i}} - 1\right). \tag{44}$$

Now, we consider the following two subcases:

Case 2a: $\bar{\sigma}_i^2 \leq \frac{64}{27}\sigma_{p,i}^2$. By (44), we have $\left[\frac{\partial G_1(\bar{\sigma}_i,\alpha_d)}{\partial \alpha_d}\right]_{\alpha_d=\frac{\sigma_{p,i}}{\sqrt{3}\bar{\sigma}_i}} \leq 0$ when $\bar{\sigma}_i^2 \leq \frac{64}{27}\sigma_{p,i}^2$. Since $\frac{\partial G_1(\bar{\sigma}_i,\alpha_d)}{\partial \alpha_d}$ is nonpositive even when it reaches its maximum value at $\alpha_d = \frac{\sigma_{p,i}}{\sqrt{3}\bar{\sigma}_i}$, we conclude that $\frac{\partial G_1(\bar{\sigma}_i,\alpha_d)}{\partial \alpha_d} \leq 0$ for all $\alpha_d \in [0,1/2]$, which means that $G_1(\bar{\sigma}_i,\alpha_d)$ decreases in α_d for $\alpha_d \in [0,1/2]$. Since $G_1(\bar{\sigma}_i,\alpha_d) > 0$ at $\alpha_d = 1/2$ by (42), we conclude that $G_1(\bar{\sigma}_i,\alpha_d) > 0$ for all $\alpha_d \in [0,1/2]$.

Case 2b: $\bar{\sigma}_i^2 > \frac{64}{27}\sigma_{p,i}^2$. Recall from Case 2a that if $\bar{\sigma}_i^2 \leq \frac{64}{27}\sigma_{p,i}^2$, $G_1(\bar{\sigma}_i,\alpha_d)$ is positive for all $\alpha_d \in [0,1/2]$. Since $G_1(\bar{\sigma}_i,\alpha_d)$ increases with $\bar{\sigma}_i$ (recall (38)), it follows that when $\bar{\sigma}_i^2 > \frac{64}{27}\sigma_{p,i}^2$, $G_1(\bar{\sigma}_i,\alpha_d)$ will also be positive for all $\alpha_d \in [0,1/2]$.

(c) Setting α_d equal to 1 in (37), the derivative of EU_i^{CP} at $\alpha_d = 1$ is:

$$\left[\frac{\partial EU_i^{CP}}{\partial \alpha_d}\right]_{\alpha_d=1} = \frac{\bar{\sigma}_i^2\sigma_{p,i}^2}{\left(\bar{\sigma}_i^2+\sigma_{p,i}^2\right)^{3/2}}\left[\frac{-\Delta\mu_i^2}{\bar{\sigma}_i^2+\sigma_{p,i}^2}+1\right]\phi\left(\frac{\Delta\mu_i}{\sqrt{\bar{\sigma}_i^2+\sigma_{p,i}^2}}\right) < 0, \tag{45}$$

where the inequality follows from $|\Delta\mu_i| > \sqrt{\bar{\sigma}_i^2+\sigma_{p,i}^2}$. Since the derivative of EU_i^{CP} at $\alpha_d = 1$ is negative by (45) and EU_i^{CP} is a quasiconcave function of α_d for $\alpha_d \in \left[1/2, \min\left\{\frac{\sigma_X^2+\sigma_{d,X}^2}{\sigma_X^2}, \frac{\sigma_Y^2+\sigma_{d,Y}^2}{\sigma_Y^2}\right\}\right]$ by part (a) of this lemma, it follows that EU_i^{CP} decreases in α_d if $\alpha_d \geq 1$.

■

Now, we are ready to prove Proposition 1 using Lemma A2.

Proof of Proposition 1: (a) The derivative of EU^{PM} in (19) with respect to α_d is:

$$\frac{\partial EU^{PM}}{\partial \alpha_d} = \frac{\Delta\mu^2}{\alpha_d^2\bar{\sigma}}\left(\frac{1}{\alpha_d}-1\right)\phi\left(\frac{\Delta\mu}{\alpha_d\bar{\sigma}}\right).$$

It is obvious that $\frac{\partial EU^{PM}}{\partial \alpha_d} > 0$ for all $\alpha_d < 1$, and $\frac{\partial EU^{PM}}{\partial \alpha_d} < 0$ for all $\alpha_d > 1$. Hence, EU^{PM} is a unomidal function, which reaches its maximum at $\alpha_d = 1$.

(b) It follows from Lemma A2(b) that if $|\Delta\mu_1| < \sqrt{\bar{\sigma}_1^2+\sigma_{p,1}^2}$, and $|\Delta\mu_2| < \sqrt{\bar{\sigma}_2^2+\sigma_{p,2}^2}$ hold, both EU_1^{CP} and EU_2^{CP} increase with α_d for all $\alpha_d \in [0,1]$. This implies that EU^{CP} given with (21) increases with α_d for all $\alpha_d \in [0,1]$, and thus, EU^{CP} reaches its maximum value when $\alpha_d > 1$, i.e., $\alpha_d^* > 1$.

Second, it follows from Lemma A2(c) that if $|\Delta\mu_1| > \sqrt{\bar{\sigma}_1^2+\sigma_{p,1}^2}$, and $|\Delta\mu_2| > \sqrt{\bar{\sigma}_2^2+\sigma_{p,2}^2}$ hold, both EU_1^{CP} and EU_2^{CP} decrease with α_d for all $\alpha_d \in \left[1, \min\left\{\frac{\sigma_X^2+\sigma_{d,X}^2}{\sigma_X^2}, \frac{\sigma_Y^2+\sigma_{d,Y}^2}{\sigma_Y^2}\right\}\right]$. This implies that EU^{CP} given with (21) decreases with α_d for all $\alpha_d \in \left[1, \min\left\{\frac{\sigma_X^2+\sigma_{d,X}^2}{\sigma_X^2}, \frac{\sigma_Y^2+\sigma_{d,Y}^2}{\sigma_Y^2}\right\}\right]$, and thus, EU^{CP} reaches its maximum value when $\alpha_d < 1$, i.e., $\alpha_d^* < 1$.

■

B.2. Proof of Section 5 Results

B.2.1. Proof of Section 5.1 Results

The derivative of EU^{PM} in (19) with respect to r_X and r_Y are equal to

$$\frac{\partial EU^{PM}}{\partial r_X} = \frac{\partial EU^{PM}}{\partial \bar{\sigma}} \frac{\partial \bar{\sigma}}{\partial r_X} = \frac{\bar{w}^2}{2\bar{\sigma}} \frac{\partial EU^{PM}}{\partial \bar{\sigma}} \text{ and } \frac{\partial EU^{PM}}{\partial r_Y} = \frac{\partial EU^{PM}}{\partial \bar{\sigma}} \frac{\partial \bar{\sigma}}{\partial r_Y} = \frac{(1-\bar{w})^2}{2\bar{\sigma}} \frac{\partial EU^{PM}}{\partial \bar{\sigma}}, \tag{46}$$

where

$$\frac{\partial EU^{PM}}{\partial \bar{\sigma}} = \left(\frac{\Delta\mu^2}{\alpha_d \bar{\sigma}^2} \left(\frac{1}{\alpha_d} - 1 \right) + 1 \right) \phi \left(\frac{\Delta\mu}{\alpha_d \bar{\sigma}} \right). \tag{47}$$

Proof of Proposition 2: When $r_X = r_Y = 0$, one can easily confirm from (18) and (19) that $EU^{PM} - EU^{SM} = 0$. Furthermore, we observe from (46) and (47) that $\frac{\partial EU^{PM}}{\partial r_X} \geq 0$ and $\frac{\partial EU^{PM}}{\partial r_Y} \geq 0$ for $\alpha_d = 1$, i.e., EU^{PM} increases with both r_X and r_Y when $\alpha_d = 1$. Since $EU^{PM} - EU^{SM} = 0$ for $r_X = r_Y = 0$, and EU^{PM} increases with r_X and r_Y for $\alpha_d = 1$, we have $EU^{PM} - EU^{SM} \geq 0$ for $r_X \geq 0$ and $r_Y \geq 0$. ■

Proof of Proposition 3: (a) When $r_X = r_Y = 0$, one can easily confirm from (18) and (19) that $EU^{PM} - EU^{SM} = 0$. Furthermore, we observe from (46) and (47) that $\frac{\partial EU^{PM}}{\partial r_X} \geq 0$ and $\frac{\partial EU^{PM}}{\partial r_Y} \geq 0$ for $\alpha_d < 1$, i.e., EU^{PM} increases with both r_X and r_Y when $\alpha_d = 1$. Since $EU^{PM} - EU^{SM} = 0$ for $r_X = r_Y = 0$, and EU^{PM} increases with r_X and r_Y for $\alpha_d < 1$, we have $EU^{PM} - EU^{SM} \geq 0$ for $r_X \geq 0$ and $r_Y \geq 0$.

(b) The proof consists of two parts: In part I, we will investigate the monotonicity/unimodality of EU^{PM} with respect to r_X and r_Y, whereas in part II, we will analyze when PM outperforms SM.

Part I: Observe from (47) that when $\alpha_d > 1$, $\frac{\partial EU^{PM}}{\partial \bar{\sigma}} < 0$ for all $\bar{\sigma} \in \left[0, \sqrt{\frac{\Delta\mu^2}{\alpha_d}\left(1 - \frac{1}{\alpha_d}\right)}\right)$, and $\frac{\partial EU^{PM}}{\partial \bar{\sigma}} > 0$ for all $\bar{\sigma} > \sqrt{\frac{\Delta\mu^2}{\alpha_d}\left(1 - \frac{1}{\alpha_d}\right)}$. Hence, EU^{PM} is a unimodal function of $\bar{\sigma}$, which reaches its minimum at $\bar{\sigma} = \sqrt{\frac{\Delta\mu^2}{\alpha_d}\left(1 - \frac{1}{\alpha_d}\right)}$. Hence, recalling the definition of $\bar{\sigma}$ in (17), it follows that for a given r_X, we have:

$$\begin{cases} EU^{PM} \text{ increases in } r_Y \text{ for all } r_Y \geq 0, & \text{if } r_X > \frac{1}{\bar{w}^2} \frac{\Delta\mu^2}{\alpha_d}\left(1 - \frac{1}{\alpha_d}\right), \\ EU^{PM} \text{ decreases in } r_Y \text{ for all } r_Y \in \left[0, \frac{1}{(1-\bar{w})^2}\left(\frac{\Delta\mu^2}{\alpha_d}\left(1 - \frac{1}{\alpha_d}\right) - \bar{w}^2 r_X\right)\right), & \\ \text{and } EU^{PM} \text{ increases in } r_Y \text{ for all } r_Y \in \left(\frac{1}{(1-\bar{w})^2}\left(\frac{\Delta\mu^2}{\alpha_d}\left(1 - \frac{1}{\alpha_d}\right) - \bar{w}^2 r_X\right), \infty\right), & \text{if } r_X < \frac{1}{\bar{w}^2} \frac{\Delta\mu^2}{\alpha_d}\left(1 - \frac{1}{\alpha_d}\right). \end{cases}$$

Similarly, for a given r_Y, we have:

$$\begin{cases} EU^{PM} \text{ increases in } r_X \text{ for all } r_X \geq 0, & \text{if } r_Y > \frac{1}{(1-\bar{w})^2} \frac{\Delta\mu^2}{\alpha_d}\left(1 - \frac{1}{\alpha_d}\right), \\ EU^{PM} \text{ decreases in } r_X \text{ for all } r_X \in \left[0, \frac{1}{\bar{w}^2}\left(\frac{\Delta\mu^2}{\alpha_d}\left(1 - \frac{1}{\alpha_d}\right) - (1-\bar{w})^2 r_Y\right)\right), & \\ \text{and } EU^{PM} \text{ increases in } r_X \text{ for all } r_X \in \left(\frac{1}{\bar{w}^2}\left(\frac{\Delta\mu^2}{\alpha_d}\left(1 - \frac{1}{\alpha_d}\right) - (1-\bar{w})^2 r_Y\right), \infty\right), & \text{if } r_Y < \frac{1}{(1-\bar{w})^2} \frac{\Delta\mu^2}{\alpha_d}\left(1 - \frac{1}{\alpha_d}\right). \end{cases}$$

Part II: One can easily confirm from (18) and (19) that when $\bar{\sigma} = 0$, we have $EU^{PM} - EU^{SM} = 0$. Furthermore, as $\bar{\sigma}$ tends to ∞, $EU^{PM} - EU^{SM}$ goes to ∞. Since EU^{PM} is a unimodal function of $\bar{\sigma}$, i.e., it first decreases, then increases in $\bar{\sigma}$ (recall part I), it follows that there exists a unique threshold ψ such that if $\bar{\sigma} < \psi$, we have $EU^{PM} < EU^{SM}$, and if

$\bar{\sigma} > \psi$, we have $EU^{PM} > EU^{SM}$. This, together with the definition of $\bar{\sigma}$ in (17), implies that for a given r_X, we have:

$$\begin{cases} EU^{PM} > EU^{SM} \text{ for all } r_Y \geq 0, & \text{if } r_X > \frac{\psi^2}{\bar{w}^2}, \\ EU^{PM} < EU^{SM} \text{ for all } r_Y \in \left[0, \frac{\psi^2 - \bar{w}^2 r_X}{(1-\bar{w})^2}\right), & \\ \text{and } EU^{PM} > EU^{SM} \text{ for all } r_Y \in \left(\frac{\psi^2 - \bar{w}^2 r_X}{(1-\bar{w})^2}, \infty\right), & \text{if } r_X < \frac{\psi^2}{\bar{w}^2}. \end{cases}$$

Similarly, for a given r_Y, we have:

$$\begin{cases} EU^{PM} > EU^{SM} \text{ for all } r_X \geq 0, & \text{if } r_Y > \frac{\psi^2}{(1-\bar{w})^2}, \\ EU^{PM} < EU^{SM} \text{ for all } r_X \in \left[0, \frac{\psi^2 - (1-\bar{w})^2 r_Y}{\bar{w}^2}\right), & \\ \text{and } EU^{PM} > EU^{SM} \text{ for all } r_X \in \left(\frac{\psi^2 - (1-\bar{w})^2 r_Y}{\bar{w}^2}, \infty\right), & \text{if } r_X < \frac{\psi^2}{(1-\bar{w})^2}. \end{cases}$$

$$\psi_X = \frac{(\psi^2 - \bar{w}^2 r_X)^+}{(1-\bar{w})^2}, \text{ and } \psi_Y = \frac{(\psi^2 - (1-\bar{w})^2 r_Y)^+}{\bar{w}^2},$$

where x^+ denotes $\max\{x, 0\}$. Using these definitions of ψ_X and ψ_Y, we conclude that for a fixed r_Y (r_X), $EU^{PM} \geq EU^{SM}$ if and only if $r_X \geq \psi_X$ ($r_Y \geq \psi_Y$). ■

B.2.2. Proof of Section 5.2 Results

For the proof of Propositions 4 and 5, we first need the following lemma to establish the structure of EU_i^{CP} given with (23) with respect to $\bar{\sigma}_i$ given with (17), r_X and r_Y.

Lemma A3 *Consider EU_i^{PC} and EU_i^{CP} given with (22) and (23), respectively. Then:*

(a) EU_i^{CP} is quasiconvex in $\bar{\sigma}_i$.

(b) If $|\Delta\mu_i| < \sqrt{2\sigma_{p,i}^2/\alpha_d}$, then (i) EU_i^{CP} increases in r_X, and r_Y, and (ii) $EU_i^{CP} \geq EU_i^{PC}$ holds for all $r_X, r_Y \geq 0$.

(c) Let $|\Delta\mu_i| > \sqrt{2\sigma_{p,i}^2/\alpha_d}$ and let τ_i denote the unique value of $\bar{\sigma}_i$ that satisfies the following equation:[6]

$$\Delta\mu_i^2 \alpha_d \left(\frac{\alpha_d \bar{\sigma}_i^2}{\alpha_d^2 \bar{\sigma}_i^2 + \sigma_{p,i}^2} - 1\right) + \alpha_d^2 \bar{\sigma}_i^2 + 2\sigma_{p,i}^2 = 0. \tag{48}$$

Then:

(i) EU_i^{CP} decreases in $\bar{\sigma}_i$ if $\bar{\sigma}_i < \tau_i$, and it increases in $\bar{\sigma}_i$ if $\bar{\sigma}_i > \tau_i$.

(ii) There exists a unique threshold η_i such that $EU_i^{CP} < EU_i^{PC}$ if $\bar{\sigma}_i < \eta_i$, and $EU_i^{CP} > EU_i^{PC}$ if $\bar{\sigma}_i > \eta_i$.

Proof of Lemma A3: (a) The derivative of EU_i^{CP} in (23) with respect to r_X and r_Y are equal to

$$\frac{\partial EU_i^{CP}}{\partial r_X} = \frac{\partial EU_i^{CP}}{\partial \bar{\sigma}_i} \frac{\partial \bar{\sigma}_i}{\partial r_X} = \frac{w_i^2}{2\bar{\sigma}_i} \frac{\partial EU_i^{CP}}{\partial \bar{\sigma}_i} \text{ and } \frac{\partial EU_i^{CP}}{\partial r_Y} = \frac{\partial EU_i^{CP}}{\partial \bar{\sigma}_i} \frac{\partial \bar{\sigma}_i}{\partial r_Y} = \frac{(1-w_i)^2}{2\bar{\sigma}} \frac{\partial EU_i^{CP}}{\partial \bar{\sigma}_i}, \tag{49}$$

where

$$\frac{\partial EU_i^{CP}}{\partial \bar{\sigma}_i} = \frac{\alpha_d \bar{\sigma}_i}{\left(\alpha_d^2 \bar{\sigma}_i^2 + \sigma_{p,i}^2\right)^{3/2}} \left[\Delta\mu_i^2 \alpha_d \left(\frac{\alpha_d \bar{\sigma}_i^2}{\alpha_d^2 \bar{\sigma}_i^2 + \sigma_{p,i}^2} - 1\right) + \alpha_d^2 \bar{\sigma}_i^2 + 2\sigma_{p,i}^2\right] \phi\left(\frac{\Delta\mu_i}{\sqrt{\alpha_d^2 \bar{\sigma}_i^2 + \sigma_{p,i}^2}}\right). \tag{50}$$

One can easily confirm in (50) that $\Delta\mu_i^2 \alpha_d \left(\frac{\alpha_d \bar{\sigma}_i^2}{\alpha_d^2 \bar{\sigma}_i^2 + \sigma_{p,i}^2} - 1\right) + \alpha_d^2 \bar{\sigma}_i^2 + 2\sigma_{p,i}^2$ increases in $\bar{\sigma}_i$, and thus, EU_i^{CP} is quasiconvex in $\bar{\sigma}_i$.

[6] We show the existence and uniqueness of τ_i in the proof.

(b) *Part (i):* First note that, if $|\Delta\mu_i| < \sqrt{\frac{2\sigma_{p,i}^2}{\alpha_d}}$ holds, $\frac{\partial EU_i^{CP}}{\partial \bar{\sigma}_i} > 0$ at $\bar{\sigma}_i = 0$. Then, EU_i^{CP} increases in $\bar{\sigma}_i$ for all $\bar{\sigma}_i \geq 0$ since EU_i^{CP} is quasiconvex in $\bar{\sigma}_i$ by part (a) of this lemma. This implies that EU_i^{CP} increases with both r_X and r_Y (recall from (17) that $\bar{\sigma}_i$ is an increasing function of both r_X and r_Y).

Part (ii): When $r_X = r_Y = 0$, one can easily confirm from (22) and (23) that $EU_i^{CP} - EU_i^{PC} = 0$. Since $EU_i^{CP} - EU_i^{PC} = 0$ for $r_X = r_Y = 0$, and EU_i^{CP} increases with r_X and r_Y as per part I, we have $EU_i^{CP} - EU_i^{PC} \geq 0$ for all $r_X \geq 0$ and $r_Y \geq 0$.

(c) *Part (i):* First note that, if $|\Delta\mu_i| > \sqrt{\frac{2\sigma_{p,i}^2}{\alpha_d}}$ holds, $\frac{\partial EU_i^{CP}}{\partial \bar{\sigma}_i} < 0$ at $\bar{\sigma}_i = 0$. On the other hand, $\frac{\partial EU_i^{CP}}{\partial \bar{\sigma}_i}$ tends to zero from positive values as $\bar{\sigma}_i$ tends to ∞. Then, by Intermediate Value Theorem, there exists a value of $\bar{\sigma}_i > 0$, denoted by τ_i, that satisfies $\frac{\partial EU_i^{CP}}{\partial \bar{\sigma}_i} = 0$, or, equivalently, (48). Moreover, since EU_i^{CP} is quasiconvex in $\bar{\sigma}_i$ by part (a) of this lemma, τ_i is unique and EU_i^{CP} decreases with $\bar{\sigma}_i$ for all $\bar{\sigma}_i \in [0, \tau_i)$, and it increases with $\bar{\sigma}_i$ for all $\bar{\sigma}_i > \tau_i$.

Part (ii): One can easily confirm from (22) and (23) that when $\bar{\sigma}_i = 0$, we have $EU_i^{CP} - EU_i^{PC} = 0$. Furthermore, as $\bar{\sigma}_i$ tends to ∞, $EU_i^{CP} - EU_i^{PC}$ goes to ∞. Since EU_i^{CP} first decreases, then increases in $\bar{\sigma}_i$ (recall part I), it follows that there exists a unique threshold η_i such that if $\bar{\sigma}_i < \eta_i$, we have $EU_i^{CP} < EU_i^{PC}$, and if $\bar{\sigma}_i > \eta_i$, we have $EU_i^{CP} > EU_i^{PC}$. ■

Now, we are ready to prove Propositions 4 and 5 using Lemma A3.

Proof of Proposition 4: First, it follows from part (i) of Lemma A3(b) that if $|\Delta\mu_1| < \sqrt{2\sigma_{p,1}^2/\alpha_d}$ and $|\Delta\mu_2| < \sqrt{2\sigma_{p,2}^2/\alpha_d}$ hold, both EU_1^{CP} and EU_2^{CP} increase with r_X and r_Y for all $r_X, r_Y \geq 0$. This implies that EU^{CP} given with (21) increases with r_X and r_Y for all $r_X, r_Y \geq 0$.

Second, it follows from part (ii) of Lemma A3(b) that if $|\Delta\mu_1| < \sqrt{2\sigma_{p,1}^2/\alpha_d}$ and $|\Delta\mu_2| < \sqrt{2\sigma_{p,2}^2/\alpha_d}$ hold, we have $EU_1^{CP} \geq EU_1^{PC}$ and $EU_2^{CP} \geq EU_2^{PC}$. Then, recalling (20) and (21), we conclude that $EU^{CP} \geq EU^{PC}$ for all $r_X, r_Y \geq 0$. ■

Proof of Proposition 5: (a) First, it follows from part (i) of Lemma A3(c) that if $|\Delta\mu_1| > \sqrt{2\sigma_{p,1}^2/\alpha_d}$ and $|\Delta\mu_2| > \sqrt{2\sigma_{p,2}^2/\alpha_d}$ hold, and if $\bar{\sigma}_1 > \tau_1$ and $\bar{\sigma}_2 > \tau_2$, EU_1^{CP} and EU_2^{CP} increase with $\bar{\sigma}_1$ and $\bar{\sigma}_2$, respectively. Recalling from (17) that both $\bar{\sigma}_1$ and $\bar{\sigma}_2$ increase with r_X and r_Y, this means that when $|\Delta\mu_1| > \sqrt{2\sigma_{p,1}^2/\alpha_d}$ and $|\Delta\mu_2| > \sqrt{2\sigma_{p,2}^2/\alpha_d}$, both EU_1^{CP} and EU_2^{CP} increase with r_X and r_Y for all $r_X, r_Y \geq 0$ satisfying $w_1^2 r_X + (1-w_1)^2 r_Y > \tau_1^2$ and $w_2^2 r_X + (1-w_2)^2 r_Y > \tau_2^2$. Then, from (21), it follows that EU^{CP} increases with r_X and r_Y for all $r_X, r_Y \geq 0$ satisfying $w_1^2 r_X + (1-w_1)^2 r_Y > \tau_1^2$ and $w_2^2 r_X + (1-w_2)^2 r_Y > \tau_2^2$, i.e., for sufficiently large r_X and r_Y.

Second, it follows from part (ii) of Lemma A3(c) that if $|\Delta\mu_1| > \sqrt{2\sigma_{p,1}^2/\alpha_d}$ and $|\Delta\mu_2| > \sqrt{2\sigma_{p,2}^2/\alpha_d}$ hold, and if $\bar{\sigma}_1 > \eta_1$ and $\bar{\sigma}_2 > \eta_2$, we have $EU_1^{CP} > EU_1^{PC}$ and $EU_2^{CP} > EU_2^{PC}$. Recalling (17), this means that when $|\Delta\mu_1| > \sqrt{2\sigma_{p,1}^2/\alpha_d}$ and $|\Delta\mu_2| > \sqrt{2\sigma_{p,2}^2/\alpha_d}$, we have $EU_1^{CP} > EU_1^{PC}$ and $EU_2^{CP} > EU_2^{PC}$ for all $r_X, r_Y \geq 0$ satisfying $w_1^2 r_X + (1-w_1)^2 r_Y > \eta_1^2$ and $w_2^2 r_X + (1-w_2)^2 r_Y > \eta_2^2$. Then, from (20) and (21), it follows that $EU^{CP} > EU^{PC}$ for all $r_X, r_Y \geq 0$ satisfying $w_1^2 r_X + (1-w_1)^2 r_Y > \tau_1^2$ and $w_2^2 r_X + (1-w_2)^2 r_Y > \tau_2^2$, i.e., for sufficiently large r_X and r_Y.

(b) First, it follows from part (i) of Lemma A3(c) that if $|\Delta\mu_1| > \sqrt{2\sigma_{p,1}^2/\alpha_d}$ and $|\Delta\mu_2| > \sqrt{2\sigma_{p,2}^2/\alpha_d}$ hold, and if $\bar{\sigma}_1 < \tau_1$ and $\bar{\sigma}_2 < \tau_2$, EU_1^{CP} and EU_2^{CP} decrease with $\bar{\sigma}_1$ and $\bar{\sigma}_2$, respectively. Recalling from (17) that both $\bar{\sigma}_1$ and $\bar{\sigma}_2$ increase with r_X and r_Y, this means that when $|\Delta\mu_1| > \sqrt{2\sigma_{p,1}^2/\alpha_d}$ and $|\Delta\mu_2| > \sqrt{2\sigma_{p,2}^2/\alpha_d}$, both EU_1^{CP} and EU_2^{CP} decrease with r_X and r_Y for all $r_X, r_Y \geq 0$ satisfying $w_1^2 r_X + (1-w_1)^2 r_Y < \tau_1^2$ and $w_2^2 r_X + (1-w_2)^2 r_Y < \tau_2^2$. Then,

from (21), it follows that EU^{CP} decreases with r_X and r_Y for all $r_X, r_Y \geq 0$ satisfying $w_1^2 r_X + (1-w_1)^2 r_Y < \tau_1^2$ and $w_2^2 r_X + (1-w_2)^2 r_Y < \tau_2^2$, i.e., for sufficiently low r_X and r_Y.

Second, it follows from part (ii) of Lemma A3(c) that if $|\Delta\mu_1| > \sqrt{2\sigma_{p,1}^2/\alpha_d}$ and $|\Delta\mu_2| > \sqrt{2\sigma_{p,2}^2/\alpha_d}$ hold, and if $\bar{\sigma}_1 < \eta_1$ and $\bar{\sigma}_2 < \eta_2$, we have $EU_1^{CP} < EU_1^{PC}$ and $EU_2^{CP} < EU_2^{PC}$. Recalling (17), this means that when $|\Delta\mu_1| > \sqrt{2\sigma_{p,1}^2/\alpha_d}$ and $|\Delta\mu_2| > \sqrt{2\sigma_{p,2}^2/\alpha_d}$, we have $EU_1^{CP} < EU_1^{PC}$ and $EU_2^{CP} < EU_2^{PC}$ for all $r_X, r_Y \geq 0$ satisfying $w_1^2 r_X + (1-w_1)^2 r_Y < \eta_1^2$ and $w_2^2 r_X + (1-w_2)^2 r_Y < \eta_2^2$. Then, from (20) and (21), it follows that $EU^{CP} < EU^{PC}$ for all $r_X, r_Y \geq 0$ satisfying $w_1^2 r_X + (1-w_1)^2 r_Y < \eta_1^2$ and $w_2^2 r_X + (1-w_2)^2 r_Y < \eta_2^2$, i.e., for sufficiently low r_X and r_Y.

(c) One can numerically check that when $p = 0.5$, $w_1 = 0.75$, $w_2 = 0.25$, $\Delta\mu_1 = 1.25$, $\Delta\mu_2 = -4.25$, $\sigma_{p,1} = \sigma_{p,2} = 1$, $\alpha_d = 1.5$, $r_X = 0.5705$, the optimal prognosis-to-treatment process is CP at $r_Y = 0.1105$, it is PC at $r_Y = 5.4424$, whereas the optimal process switches back to CP at $r_Y = 20.1038$. ■

B.3. Proof of Section 6 Results

For the proof of Proposition 6, we first need the following lemma to establish the monotonicity of EU^{PC} and EU^{CP} in (21) and (20) with respect to $\sigma_{p,1}$ and $\sigma_{p,2}$.

Lemma A4 *(a)* $\frac{\partial EU^{PC}}{\partial \sigma_{p,1}} < 0$ *and* $\frac{\partial EU^{PC}}{\partial \sigma_{p,2}} < 0$ *for all* $\sigma_{p,1}, \sigma_{p,2} \geq 0$.

(b) For $\frac{\partial EU^{CP}}{\partial \sigma_{p,1}}$ *and* $\frac{\partial EU^{CP}}{\partial \sigma_{p,2}}$*, we have:*

(i) If $\alpha_d \geq 1$ *or if* $\alpha_d < 1$ *and* $\Delta\mu_1^2 \leq \frac{\alpha_d^2}{1-\alpha_d}\bar{\sigma}_1^2$*, we have* $\frac{\partial EU^{CP}}{\partial \sigma_{p,1}} < 0$ *for all* $\sigma_{p,1} \geq 0$*, whereas if* $\alpha_d < 1$ *and* $\Delta\mu_1^2 > \frac{\alpha_d^2}{1-\alpha_d}\bar{\sigma}_1^2$*, we have* $\frac{\partial EU^{CP}}{\partial \sigma_{p,1}} < 0$ *if and only if* $\sigma_{p,1} \geq \sqrt{\alpha_d \bar{\sigma}_1^2 \frac{\Delta\mu_1^2}{\Delta\mu_1^2 + \alpha_d \bar{\sigma}_1^2} - \alpha_d^2 \bar{\sigma}_1^2}$.

(ii) If $\alpha_d \geq 1$ *or if* $\alpha_d < 1$ *and* $\Delta\mu_2^2 \leq \frac{\alpha_d^2}{1-\alpha_d}\bar{\sigma}_2^2$*, we have* $\frac{\partial EU^{CP}}{\partial \sigma_{p,2}} < 0$ *for all* $\sigma_{p,2} \geq 0$*, whereas if* $\alpha_d < 1$ *and* $\Delta\mu_2^2 > \frac{\alpha_d^2}{1-\alpha_d}\bar{\sigma}_2^2$*, we have* $\frac{\partial EU^{CP}}{\partial \sigma_{p,2}} < 0$ *if and only if* $\sigma_{p,2} \geq \sqrt{\alpha_d \bar{\sigma}_2^2 \frac{\Delta\mu_2^2}{\Delta\mu_2^2 + \alpha_d \bar{\sigma}_2^2} - \alpha_d^2 \bar{\sigma}_2^2}$.

Proof of Lemma A4: (a) The derivative of EU^{PC} with respect to $\sigma_{p,1}$ and $\sigma_{p,2}$ are:

$$\frac{\partial EU^{PC}}{\partial \sigma_{p,1}} = -\frac{\Delta\mu_1^2}{\sigma_{p,1}^2}\phi\left(\frac{\Delta\mu_1}{\sigma_{p,1}}\right) < 0, \text{ and } \frac{\partial EU^{PC}}{\partial \sigma_{p,2}} = -\frac{\Delta\mu_2^2}{\sigma_{p,2}^2}\phi\left(\frac{\Delta\mu_2}{\sigma_{p,2}}\right) < 0.$$

(b) We will prove only part (i). The proof of part (ii) follows the same lines and thus, we skip it. The derivative of EU^{CP} with respect to $\sigma_{p,1}$ is:

$$\frac{\partial EU^{CP}}{\partial \sigma_{p,1}} = \frac{\sigma_{p,1}}{\left(\alpha_d^2 \bar{\sigma}_1^2 + \sigma_{p,1}^2\right)^{3/2}} \left[\Delta\mu_1^2 \left(\frac{\alpha_d \bar{\sigma}_1^2}{\alpha_d^2 \bar{\sigma}_1^2 + \sigma_{p,1}^2} - 1\right) - \alpha_d \bar{\sigma}_1^2\right] \phi\left(\frac{\Delta\mu_1}{\sqrt{\alpha_d^2 \bar{\sigma}_1^2 + \sigma_{p,1}^2}}\right). \tag{51}$$

We consider the following two cases:

Case 1: $\alpha_d \geq 1$. One can easily confirm in (51) that when $\alpha_d \geq 1$, $\frac{\partial EU^{CP}}{\partial \sigma_{p,1}} < 0$ holds for all $\sigma_{p,1} \geq 0$.

Case 2: $\alpha_d < 1$. One can easily confirm in (51) that $\Delta\mu_1^2 \left(\frac{\alpha_d \bar{\sigma}_1^2}{\alpha_d^2 \bar{\sigma}_1^2 + \sigma_{p,1}^2} - 1\right) - \alpha_d \bar{\sigma}_1^2$ decreases in $\sigma_{p,1}$, and thus, EU^{CP} is quasiconcave in $\sigma_{p,1}$. Now, we consider the following two subcases:

Case 2a: $\Delta\mu_1^2 \leq \frac{\alpha_d^2}{1-\alpha_d}\bar{\sigma}_1^2$. First note that, if $\Delta\mu_1^2 \leq \frac{\alpha_d^2}{1-\alpha_d}\bar{\sigma}_1^2$ holds, $\frac{\partial EU^{CP}}{\partial \sigma_{p,1}} \leq 0$ at $\sigma_{p,1} = 0$. Then, EU^{CP} decreases in $\sigma_{p,1}$ for all $\sigma_{p,1} \geq 0$ since EU^{CP} is quasiconcave in $\sigma_{p,1}$.

Case 2b: $\Delta\mu_1^2 > \frac{\alpha_d^2}{1-\alpha_d}\bar{\sigma}_1^2$. First note that, if $\Delta\mu_1^2 > \frac{\alpha_d^2}{1-\alpha_d}\bar{\sigma}_1^2$ holds, $\frac{\partial EU^{CP}}{\partial \sigma_{p,1}} > 0$ at $\sigma_{p,1} = 0$. Furthermore, $\frac{\partial EU^{CP}}{\partial \sigma_{p,1}}$ tends to zero from negative values as $\sigma_{p,1}$ tends to ∞. Then, since EU^{CP} is quasiconcave in $\sigma_{p,1}$, noting that $\frac{\partial EU^{CP}}{\partial \sigma_{p,1}}$ becomes equal to zero at $\sigma_{p,1} = \sqrt{\alpha_d \bar{\sigma}_1^2 \frac{\Delta\mu_1^2}{\Delta\mu_1^2 + \alpha_d \bar{\sigma}_1^2} - \alpha_d^2 \bar{\sigma}_1^2}$, we conclude that EU^{CP} increases with $\sigma_{p,1}$ for all $\sigma_{p,1} \in \left[0, \sqrt{\alpha_d \bar{\sigma}_1^2 \frac{\Delta\mu_1^2}{\Delta\mu_1^2 + \alpha_d \bar{\sigma}_1^2} - \alpha_d^2 \bar{\sigma}_1^2}\right)$, and it decreases with $\sigma_{p,1}$ for all $\sigma_{p,1} > \sqrt{\alpha_d \bar{\sigma}_1^2 \frac{\Delta\mu_1^2}{\Delta\mu_1^2 + \alpha_d \bar{\sigma}_1^2} - \alpha_d^2 \bar{\sigma}_1^2}$.

■

We also need to establish the monotonicity of EU^{PC} and EU^{CP} in (21) and (20) with respect to d_p.

Lemma A5 *Let us set* $\beta_1 = \frac{\alpha_d \bar{\sigma}_1^2}{\alpha_d^2 \bar{\sigma}_1^2 + \sigma_{p,1}^2}$, *and* $\beta_2 = \frac{\alpha_d \bar{\sigma}_2^2}{\alpha_d^2 \bar{\sigma}_2^2 + \sigma_{p,2}^2}$. *Then:*

(a) EU^{PC} *increases in* d_p.

(b) EU^{CP} *increases in* d_p *if we have either (i)* $\beta_1 \leq 2$, *and* $\beta_2 \leq 2$, *or (ii)* $\beta_1 > 2$, *or* $\beta_2 > 2$, *and* d_p *is sufficiently large.*

Proof of Lemma A5: We introduce a new parameter $a \geq 0$, and express $\mu_{XA} - \mu_{XB}$ and $\mu_{YA} - \mu_{YB}$ in terms of a in the following form:

$$\mu_{XA} - \mu_{XB} = \Delta\mu + \frac{a}{\bar{w}}, \text{ and } \mu_{YA} - \mu_{YB} = \mu - \frac{a}{1-\bar{w}}, \tag{52}$$

where $\bar{w}$ is given with (2) and $\Delta\mu$ is given with (16). Since, $\Delta\mu < 0$, $\mu_{XA} > \mu_{XB}$ and $\mu_{YA} < \mu_{YB}$ by assumption, we need to have $\Delta\mu + \frac{a}{\bar{w}} > 0 \Leftrightarrow a > -\bar{w}\Delta\mu$. Replacing $\mu_{XA} - \mu_{XB}$ and $\mu_{YA} - \mu_{YB}$ in (3) and (4) with the expressions in (52), $\Delta\mu_1$, $\Delta\mu_2$ and d_p could be written in terms of $\Delta\mu$ and a as:

$$\Delta\mu_1 = \Delta\mu + \frac{(w_1 - w_2)(1-p)}{\bar{w}(1-\bar{w})}a, \;\; \Delta\mu_2 = \Delta\mu - \frac{(w_1 - w_2)p}{\bar{w}(1-\bar{w})}a, \text{ and } d_p = \frac{(w_1 - w_2)}{\bar{w}(1-\bar{w})}a. \tag{53}$$

Since we have $\Delta\mu_1 > 0$ and $\Delta\mu_2 < 0$ by assumption, we need to have $\Delta\mu + \frac{(w_1-w_2)(1-p)}{\bar{w}(1-\bar{w})}a > 0 \Leftrightarrow a > \frac{-\bar{w}(1-\bar{w})\Delta\mu}{(w_1-w_2)(1-p)}$. Note that we analyze the effect of a change in d_p by varying a while keeping p, w_1, w_2, and $\bar{w}$ constant.

(a) When we replace $\Delta\mu_1$ and $\Delta\mu_2$ in EU^{PC} in (20) with the expressions in (53), and take the derivative of EU^{PC} with respect to a, we have:

$$\frac{\partial EU^{PC}}{\partial a} = \frac{p(1-p)(w_1 - w_2)}{\bar{w}(1-\bar{w})}\left[\Phi\left(\frac{\Delta\mu_1}{\sigma_{p,1}}\right) + \frac{\Delta\mu_1}{\sigma_{p,1}}\phi\left(\frac{\Delta\mu_1}{\sigma_{p,1}}\right) - \Phi\left(\frac{\Delta\mu_2}{\sigma_{p,2}}\right) - \frac{\Delta\mu_2}{\sigma_{p,2}}\phi\left(\frac{\Delta\mu_2}{\sigma_{p,2}}\right)\right] \geq 0,$$

where $\Delta\mu_1$ and $\Delta\mu_2$ are given with (53), and the inequality follows from $\Delta\mu_2 < 0 < \Delta\mu_1$.

(b) The derivative of EU^{CP} in (21) with respect to a is:

$$\frac{\partial EU^{CP}}{\partial a} = \frac{p(1-p)(w_1 - w_2)}{\bar{w}(1-\bar{w})}\left[\left(\Phi\left(\frac{\Delta\mu_1}{\sqrt{\alpha_d^2\bar{\sigma}_1^2 + \sigma_{p,1}^2}}\right) + \frac{\Delta\mu_1}{\sqrt{\alpha_d^2\bar{\sigma}_1^2 + \sigma_{p,1}^2}}\left(1 - \frac{\alpha_d\bar{\sigma}_1^2}{\alpha_d^2\bar{\sigma}_1^2 + \sigma_{p,1}^2}\right)\phi\left(\frac{\Delta\mu_1}{\sqrt{\alpha_d^2\bar{\sigma}_1^2 + \sigma_{p,1}^2}}\right)\right)\right.$$
$$\left. - \left(\Phi\left(\frac{\Delta\mu_2}{\sqrt{\alpha_d^2\bar{\sigma}_2^2 + \sigma_{p,2}^2}}\right) + \frac{\Delta\mu_2}{\sqrt{\alpha_d^2\bar{\sigma}_2^2 + \sigma_{p,2}^2}}\left(1 - \frac{\alpha_d\bar{\sigma}_2^2}{\alpha_d^2\bar{\sigma}_2^2 + \sigma_{p,2}^2}\right)\phi\left(\frac{\Delta\mu_2}{\sqrt{\alpha_d^2\bar{\sigma}_2^2 + \sigma_{p,2}^2}}\right)\right)\right],$$

where $\Delta\mu_1$ and $\Delta\mu_2$ are given with (53). Setting $z_1 = \frac{\Delta\mu_1}{\sqrt{\alpha_d^2\bar{\sigma}_1^2 + \sigma_{p,1}^2}}$ and $z_2 = \frac{\Delta\mu_2}{\sqrt{\alpha_d^2\bar{\sigma}_2^2 + \sigma_{p,2}^2}}$, and recalling that $\beta_1 = \frac{\alpha_d\bar{\sigma}_1^2}{\alpha_d^2\bar{\sigma}_1^2 + \sigma_{p,1}^2}$, and $\beta_2 = \frac{\alpha_d\bar{\sigma}_2^2}{\alpha_d^2\bar{\sigma}_2^2 + \sigma_{p,2}^2}$, we can rewrite $\frac{\partial EU^{CP}}{\partial a}$ as:

$$\frac{\partial EU^{CP}}{\partial a} = \frac{p(1-p)(w_1 - w_2)}{\bar{w}(1-\bar{w})}\Big[H(z_1, \beta_1) - H(z_2, \beta_2)\Big], \tag{54}$$

where $H(z, \beta)$ is equal to:

$$H(z, \beta) = \Phi(z) + z(1-\beta)\phi(z). \tag{55}$$

To investigate the sign of $\frac{\partial EU^{CP}}{\partial a}$ we consider the following three cases:

Case 1: $\beta_1 \leq 1$ *and* $\beta_2 \leq 1$. Since $\Delta\mu_2 < 0 < \Delta\mu_1$, one can directly confirm in (55) and (54) that we have $\frac{\partial EU^{CP}}{\partial a} > 0$.

Case 2: $\beta_1 \leq 2$, $\beta_2 \leq 2$, *and* $\beta_1 > 1$ *or* $\beta_2 > 1$. First, note that when $\alpha_d \geq 0.5$, $\beta_1 \leq 2$ and $\beta_2 \leq 2$ immediately holds, i.e., $\alpha_d \geq 0.5$ is a sufficient condition for $\beta_1 \leq 2$ and $\beta_2 \leq 2$. Without loss of generality, assume that $\beta_1 < \beta_2$. Then, $\frac{\partial EU^{CP}}{\partial a}$ in (54) could be rewritten as:

$$\frac{\partial EU^{CP}}{\partial a} = \frac{p(1-p)(w_1 - w_2)}{\bar{w}(1-\bar{w})}\Big[H(z_1, \beta_2) - H(z_2, \beta_2) + z_1(\beta_2 - \beta_1)\phi(z_1)\Big]. \tag{56}$$

Since for $1 < \beta \leq 2$, $H(z,\beta)$ increases in z as per Claim 1(a) proved at the end of this lemma and $z_2 < 0 < z_1$ holds, we have $H(z_1,\beta_2) > H(z_2,\beta_2)$. Then, in (56), $\frac{\partial EU^{CP}}{\partial a} > 0$ follows (recall that $\beta_2 > \beta_1$ and $z_1 > 0$).

Case 3: $\beta_1 > 2$ *or* $\beta_2 > 2$. Without loss of generality, we again assume that $\beta_1 < \beta_2$. Then, $\frac{\partial EU^{CP}}{\partial a}$ in (54) could be rewritten as:

$$\frac{\partial EU^{CP}}{\partial a} = \frac{p(1-p)(w_1-w_2)}{\bar{w}(1-\bar{w})}\Big[(H(z_1,\beta_2)-1/2)-(H(z_2,\beta_2)-1/2)+z_1(\beta_2-\beta_1)\phi(z_1)\Big]. \tag{57}$$

As per part (ii) of Claim 1(b) proved at the end of this lemma, for a given $\beta > 2$, $H(z,\beta) - 1/2 \geq 0$ for $z \geq \zeta_2(\beta)$, i.e., for sufficiently large z, whereas $H(z,\beta) - 1/2 < 0$ for $z < \zeta_1(\beta)$, i.e., for sufficiently low z. Then, if $z_1 \geq \zeta_2(\beta_2)$ and $z_2 \leq \zeta_1(\beta_2)$, i.e., a is sufficiently large, we have $H(z_1,\beta_2) - 1/2 > 0 > H(z_2,\beta_2) - 1/2$, and thus, in (57), $\frac{\partial EU^{CP}}{\partial a} > 0$ follows (recall that $\beta_2 > \beta_1$ and $z_1 > 0$).

Claim 1: Consider $H(z,\beta)$ given with (55).

(a) For a given $\beta \in (1,2]$, $H(z,\beta)$ increases in z for all $z \in \mathbb{R}$.

(b) For a given $\beta \in (2,\infty)$, we have:

(i) $H(z,\beta)$ increases in z for all $z \in \left(-\infty, -\sqrt{\frac{\beta-2}{\beta-1}}\right] \cup \left[\sqrt{\frac{\beta-2}{\beta-1}}, \infty\right)$, whereas it decreases in z for all $z \in \left(-\sqrt{\frac{\beta-2}{\beta-1}}, \sqrt{\frac{\beta-2}{\beta-1}}\right)$.

(ii) There exist two threshold $\zeta_1(\beta)$ and $\zeta_2(\beta)$ with $\zeta_1(\beta) < 0 < \zeta_2(\beta)$ such that $H(z,\beta) - 1/2 < 0$ for all $z \in (-\infty, \zeta_1(\beta)) \cup (0, \zeta_2(\beta))$ and $H(z,\beta) - 1/2 \geq 0$ for all $z \in [\zeta_1(\beta), 0] \cup [\zeta_2(\beta), \infty)$.

Proof: The derivative of $H(z,\beta)$ with respect to z is:

$$\frac{\partial H(z,\beta)}{\partial z} = \Big[2 - \beta - z^2(1-\beta)\Big]\phi(z). \tag{58}$$

(a) One can easily confirm in (58) that when $1 < \beta \leq 2$, $\frac{\partial H(z,\beta)}{\partial z} \geq 0$ for all $z \in \mathbb{R}$.

(b) *Part (i):* It is straightforward to establish from (58) that the roots of $\frac{\partial H(z,\beta)}{\partial z}$ are $z = -\sqrt{\frac{\beta-2}{\beta-1}}$ and $z = \sqrt{\frac{\beta-2}{\beta-1}}$. Furthermore, as z tends to $-\infty$ or ∞, $H(z,\beta)$ goes to nonnegative values. Hence, $\frac{\partial H(z,\beta)}{\partial z} > 0$ for all $z < -\sqrt{\frac{\beta-2}{\beta-1}}$ and $z > \sqrt{\frac{\beta-2}{\beta-1}}$, whereas $\frac{\partial H(z,\beta)}{\partial z} < 0$ for all z with $-\sqrt{\frac{\beta-2}{\beta-1}} < z < \sqrt{\frac{\beta-2}{\beta-1}}$.

Part (ii): First, note that $H(z,\beta) - 1/2$ goes to -1/2 as z tends to $-\infty$, it goes to 1/2 as z tends to ∞, whereas it is equal to 0 at $z = 0$. Furthermore, part (i) implies that for $z \in (-\infty, 0]$, $H(z,\beta) - 1/2$ first increases, then decreases in z, whereas for $z \in [0,\infty)$, $H(z,\beta) - 1/2$ first decreases, then increases in z. Thus, for a given $\beta > 2$, there exist two threshold $\zeta_1(\beta)$ and $\zeta_2(\beta)$ with $\zeta_1(\beta) < 0 < \zeta_2(\beta)$ such that $H(z,\beta) - 1/2 < 0$ for all $z \in (-\infty, \zeta_1(\beta)) \cup (0, \zeta_2(\beta))$ and $H(z,\beta) - 1/2 \geq 0$ for all $z \in [\zeta_1(\beta), 0] \cup [\zeta_2(\beta), \infty)$.

■

Now, we are ready to prove Proposition 6 using Lemmas A4 and A5.

Proof of Proposition 6: (a) Let $\sigma_{p,1}$ and $\sigma_{p,2}$ be fixed. First, note that $EU^{PC} - EU^{SM}$ tends to ∞ as d_p goes to infinity. Since EU^{PC} increases in d_p as per Lemma A5(a), we conclude that there exists a unique nonnegative threshold $f(\sigma_{p,1}, \sigma_{p,2})$ such that $EU^{PC} - EU^{SM} > 0$ if and only if $d_p > f(\sigma_{p,1}, \sigma_{p,2})$. In particular, $f(\sigma_{p,1}, \sigma_{p,2})$ is the value of d_p which satisfies

$$EU^{PC} - EU^{SM} = 0. \tag{59}$$

Applying implicit differentiation to (59), the derivative of $f(\sigma_{p,1},\sigma_{p,2})$ with respect to $\sigma_{p,1}$ is:

$$\frac{\partial f(\sigma_{p,1},\sigma_{p,2})}{\partial \sigma_{p,1}} = -\frac{\frac{\partial (EU^{PC}-EU^{SM})}{\partial \sigma_{p,1}}}{\frac{\partial (EU^{PC}-EU^{SM})}{\partial d_p}} > 0, \text{ for all } d_p,\sigma_{p,1},\sigma_{p,2} \geq 0,$$

where the inequality follows since $\frac{\partial (EU^{PC}-EU^{SM})}{\partial \sigma_{p,1}} < 0$ as per Lemma A4(a), and $\frac{\partial (EU^{PC}-EU^{SM})}{\partial d_p} > 0$ as per LemmaA5(a). Hence, Using t$f(\sigma_{p,1},\sigma_{p,2})$ increases in $\sigma_{p,1}$. he same discussion, one can also show that $\frac{\partial f(\sigma_{p,1},\sigma_{p,2})}{\partial \sigma_{p,2}} > 0$ holds for all $\sigma_{p,2} \geq 0$.

(b) Let $\sigma_{p,1}$ and $\sigma_{p,2}$ be fixed. First, note that $EU^{CP} - EU^{PM}$ tends to ∞ as d_p goes to infinity. We consider the following four cases:

Case 1: $\alpha_d \geq 1$. Note that when $\alpha_d \geq 1$, we have $\beta_1 = \frac{\alpha_d \bar{\sigma}_1^2}{\alpha_d^2 \bar{\sigma}_1^2 + \sigma_{p,1}^2} \leq 1$, and $\beta_2 = \frac{\alpha_d \bar{\sigma}_2^2}{\alpha_d^2 \bar{\sigma}_2^2 + \sigma_{p,2}^2} \leq 1$. Since when $\beta_1,\beta_2 \leq 2$, EU^{CP} increases in d_p as per part (i) of Lemma A5(b), we conclude that there exists a unique nonnegative threshold $g(\sigma_{p,1},\sigma_{p,2})$ such that $EU^{CP} - EU^{PM} > 0$ if and only if $d_p > g(\sigma_{p,1},\sigma_{p,2})$. Furthermore, since when $\alpha_d \geq 1$, EU^{CP} decreases in $\sigma_{p,1}$ and $\sigma_{p,2}$ for all $\sigma_{p,1},\sigma_{p,2} \geq 0$ as per Lemma A4(b), $g(\sigma_{p,1},\sigma_{p,2})$ increases in $\sigma_{p,1}$ and $\sigma_{p,2}$ for all $\sigma_{p,1},\sigma_{p,2} \geq 0$.

Case 2: $0.5 \leq \alpha_d < 1$. Note that when $\alpha_d \geq 0.5$, we have $\beta_1 = \frac{\alpha_d \bar{\sigma}_1^2}{\alpha_d^2 \bar{\sigma}_1^2 + \sigma_{p,1}^2} \leq 2$, and $\beta_2 = \frac{\alpha_d \bar{\sigma}_2^2}{\alpha_d^2 \bar{\sigma}_2^2 + \sigma_{p,2}^2} \leq 2$. Since when $\beta_1,\beta_2 \leq 2$, EU^{CP} increases in d_p as per part (i) of Lemma A5(b), we conclude that there exists a unique nonnegative threshold $g(\sigma_{p,1},\sigma_{p,2})$ such that $EU^{CP} - EU^{PM} > 0$ if and only if $d_p > g(\sigma_{p,1},\sigma_{p,2})$. Furthermore, when $\alpha_d < 1$, as per Lemma A4(b), EU^{CP} decreases in $\sigma_{p,1}$ and $\sigma_{p,2}$ if and only if $\sigma_{p,1}$ and $\sigma_{p,2}$ are sufficiently large.[7] Hence, we conclude that $g(\sigma_{p,1},\sigma_{p,2})$ increases in $\sigma_{p,1}$ and $\sigma_{p,2}$ if and only if $\sigma_{p,1}$ and $\sigma_{p,2}$ are large enough.

Case 3: $0 \leq \alpha_d < 0.5$ *and* $\beta_1 = \frac{\alpha_d \bar{\sigma}_1^2}{\alpha_d^2 \bar{\sigma}_1^2 + \sigma_{p,1}^2} \leq 2$, *and* $\beta_2 = \frac{\alpha_d \bar{\sigma}_2^2}{\alpha_d^2 \bar{\sigma}_2^2 + \sigma_{p,2}^2} \leq 2$. In this case, we reach the same conclusion as Case 2, i.e., there exists a unique nonnegative threshold $g(\sigma_{p,1},\sigma_{p,2})$ such that $EU^{CP} - EU^{PM} > 0$ if and only if $d_p > g(\sigma_{p,1},\sigma_{p,2})$. Furthermore, $g(\sigma_{p,1},\sigma_{p,2})$ increases in $\sigma_{p,1}$ and $\sigma_{p,2}$ if and only if $\sigma_{p,1}$ and $\sigma_{p,2}$ are large enough.

Case 4: $0 \leq \alpha_d < 0.5$ *and* $\beta_1 = \frac{\alpha_d \bar{\sigma}_1^2}{\alpha_d^2 \bar{\sigma}_1^2 + \sigma_{p,1}^2} > 2$, *or* $\beta_2 = \frac{\alpha_d \bar{\sigma}_2^2}{\alpha_d^2 \bar{\sigma}_2^2 + \sigma_{p,2}^2} > 2$. Since when $\beta_1 > 2$ or $\beta_2 > 2$, EU^{CP} increases in d_p for sufficiently large d_p as per part (ii) of Lemma A5(b), we conclude that there exists a nonnegative threshold $g(\sigma_{p,1},\sigma_{p,2})$ such that $EU^{CP} - EU^{PM} > 0$ if $d_p > g(\sigma_{p,1},\sigma_{p,2})$. Furthermore, when $\alpha_d < 1$, as per Lemma A4(b), EU^{CP} decreases in $\sigma_{p,1}$ and $\sigma_{p,2}$ if and only if $\sigma_{p,1}$ and $\sigma_{p,2}$ are sufficiently large.[7] Hence, we conclude that $g(\sigma_{p,1},\sigma_{p,2})$ increases in $\sigma_{p,1}$ and $\sigma_{p,2}$ if and only if $\sigma_{p,1}$ and $\sigma_{p,2}$ are large enough. ■

B.4. Proof of Section 7 Results

Proof of Corollary 1: (a) It follows directly from Proposition 3(b) and Proposition 6.

(b) It follows directly from Proposition 1(a), Proposition 2, and Proposition 6.

[7] In particular, if $\Delta\mu_1^2 \leq \frac{\alpha_d^2}{1-\alpha_d}\bar{\sigma}_1^2$ $\left(\Delta\mu_2^2 \leq \frac{\alpha_d^2}{1-\alpha_d}\bar{\sigma}_2^2\right)$, EU^{CP} decreases in $\sigma_{p,1}$ $(\sigma_{p,2})$ for all $\sigma_{p,1} \geq 0$ $(\sigma_{p,2} \geq 0)$, whereas if $\Delta\mu_1^2 > \frac{\alpha_d^2}{1-\alpha_d}\bar{\sigma}_1^2$ $\left(\Delta\mu_2^2 > \frac{\alpha_d^2}{1-\alpha_d}\bar{\sigma}_2^2\right)$, EU^{CP} decreases in $\sigma_{p,1}$ $(\sigma_{p,2})$ for all $\sigma_{p,1} \geq \sqrt{\alpha_d\bar{\sigma}_1^2\frac{\Delta\mu_1^2}{\Delta\mu_1^2+\alpha_d\bar{\sigma}_1^2} - \alpha_d^2\bar{\sigma}_1^2}$ $(\sigma_{p,2} \geq \sqrt{\alpha_d\bar{\sigma}_2^2\frac{\Delta\mu_2^2}{\Delta\mu_2^2+\alpha_d\bar{\sigma}_2^2} - \alpha_d^2\bar{\sigma}_2^2}$.

(c) As per Proposition 6, when d_p is large relative to $\sigma_{p,1}$ and $\sigma_{p,2}$, patient participation is beneficial, i.e., PC and CP outperform SM and PM. Hence, for high d_p values, it is enough to compare PC and CP. Recall from (15) that high d_p implies high $|\Delta\mu_1|$ and $|\Delta\mu_2|$. And, when $|\Delta\mu_1|$ and $|\Delta\mu_2|$ are large, PC is most valuable relative to CP for $\alpha_d > 1$ as per Proposition 1(b), and for moderate r_X and r_Y as per Proposition 5(b).

(d) As per Proposition 6, when d_p is large relative to $\sigma_{p,1}$ and $\sigma_{p,2}$, patient participation is beneficial, i.e., PC and CP outperform SM and PM. Hence, for high d_p values, it is enough to compare PC and CP. Recall from (15) that high d_p implies high $|\Delta\mu_1|$ and $|\Delta\mu_2|$. And, when $|\Delta\mu_1|$ and $|\Delta\mu_2|$ are large, CP is most valuable relative to PC for $\alpha_d < 1$ as per Proposition 1(b), and for high r_X and r_Y as per Proposition 5(a).

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